Linear Algebra and Computation

What is a vector?

$$\begin{pmatrix} 5 \\ 7 \\ 12 \\ 4 \end{pmatrix} \longleftarrow \text{that, duh.}$$

Too narrow, really.

The real question is what we want from a vector.

What do we want to do with it?

Definition: A set V is called a vector space iff

- $v + w \in V$ for $v, w \in V$
- $\alpha v \in V$ for $v \in V$

α a real or complex number

with sane rules for arithmetic (associative, distributive, etc.)

What on earth are you talking about?

These are just rules--lots of things conform to these rules.

Object-oriented programming can be a little like that:

```
interface Vector {

   Vector add(Vector x, Vector y)
   Vector scale(Number alpha, Vector x)

};
```

So, is a plain old number a vector, too?

What do we need to test?

Can we add? ✓

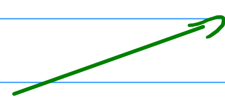
Can we multiply by a number? ✓

Are the rules of arithmetic sane? ✓

What else can be viewed a vector?

n-tuples of real numbers: $\begin{pmatrix} 5 \\ 4 \\ 7 \\ 12 \end{pmatrix}$ ← called $\mathbb{R}^n$

n-dimensional arrays of numbers

arrows  ... how?

images Demo

shapes Demo

sounds Demo

So how are all these things similar?

They have a notion of "addition" that is consistent with "normal" arithmetic.

They can be multiplied by "scalars".

(That operation is called "scaling" a vector.)

What are some useful things one can do in a vector space?

Linear combinations: $\alpha\vec{x} + \beta\vec{y} + \gamma\vec{z}$

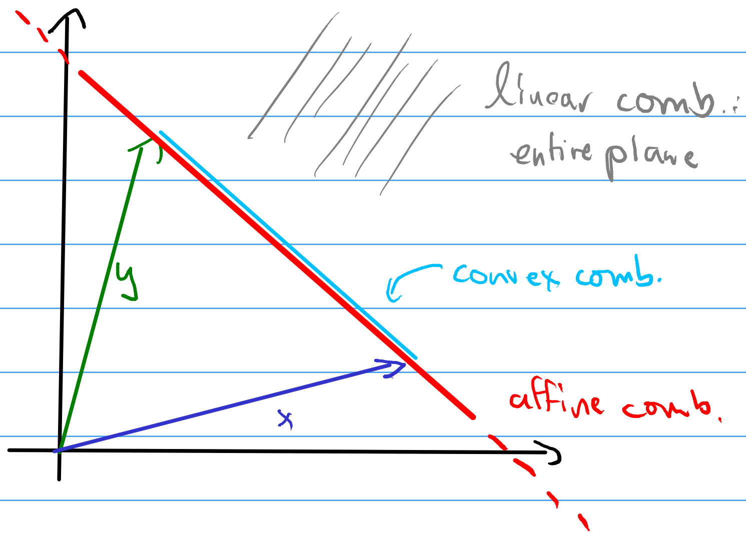
Affine combinations: ... where $\alpha+\beta+\gamma=1$

Convex combinations: ... and where $\alpha, \beta, \gamma \geq 0$

increasingly restrictive

What do these look like visualized in the plane?

(if we allow *all* possible combinations of each type)



If vectors can be arbitrarily weird, can we still describe them with numbers?

If someone gives us a basis, then we can write coordinates with respect to that basis:

basis: $\vec{x}_1, \vec{x}_2, \dots, \vec{x}_n$

coordinates: $\alpha_1, \alpha_2, \dots, \alpha_n$ $\leftarrow$ $\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{pmatrix}$ coordinate vector

linear combination: $\alpha_1 x_1 + \alpha_2 x_2 + \dots + \alpha_n x_n$

⚠ only meaningful if basis known

Can we use coordinates to describe interesting operations on vectors?

Yes, matrices describe linear functions on vector spaces.

how many times the third coordinate ends up in the first coordinate of the result

$$\begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} 2\alpha_3 \\ 0 \\ 0 \end{pmatrix}$$

dot prod.

$$\begin{array}{cc} & \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 2\alpha_3 \\ 0 \\ 0 \end{pmatrix} \end{array}$$

So what can matrices do?

Transform geometry   Demo

Traverse graphs   Demo

Blur images   Demo