CS 357: Numerical Methods

Lecture 15:
Singular Value Decomposition (SVD)
SVD Applications

Eric Shaffer

But First…More Eigenvalues

Markov Chains

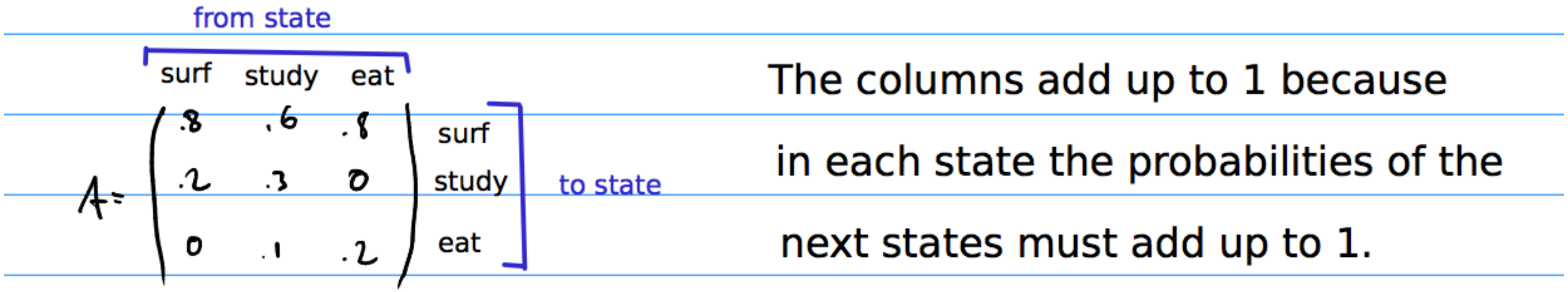
Describe a discrete system of states and transitions

(it’s a graph)

The Markov Property:
Only the current state matters in determining the probability of moving to another state

Markov Chains

- m_{ij} is the probability of moving from state i to state j



$$A=\begin{array}{c|ccc|}
 & \text{surf} & \text{study} & \text{eat} \\
\hline
\text{surf} & .8 & .6 & .8 \\
\text{study} & .2 & .3 & 0 \\
\text{eat} & 0 & .1 & .2
\end{array}$$

The columns add up to 1 because in each state the probabilities of the next states must add up to 1.

State Transitions

Modeled by a vector matrix product.

In each vector $<v_1,v_2,\ldots,v_n>$, v_i indicates the probability of being in state i

$$A\underset{\text{surf}}{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}} = \begin{pmatrix} .8 \\ .2 \\ 0 \end{pmatrix} \qquad A\begin{pmatrix} .5 \\ .5 \\ 0 \end{pmatrix} = \begin{pmatrix} .4 + .3 \\ .1 + .15 \\ .05 \end{pmatrix}$$

Equilibrium

- Equilibrium is achieved when the probabilities do not change when we compute Av

  In other words $Av = \lambda v$

Computing the SVD

Compute the eigenvalues and eigenvectors of ATA

Use Orthogonal iteration

Construct V using the eigenvectors as column vectors

Construct Σ using square roots of the eigenvalues

Find U from $A=U \Sigma V^T$

Computing the SVD

The Outer Product

What is uv^T ?

Mathematically, vectors are thought of as nx1

Another way of expressing $A = U \Sigma V^T$

What do the Singular Values Mean?

Computing the Condition Number

Rank-k Approximations to A

Rank-k Approximations to A

The Froebinius Norm*

* Not really a matrix norm