Numerical Methods (CS 357)

Worksheet

Part 1. Objectives

- **Long-term goal:** Fitting a model to data using least squares (getting close)
- Solving a least-squares problem using QR
- Setting up coefficient fitting as a least-squares problem

Part 2. Suitable Models for Linear Least-Squares

For which of the following models can you find the coefficients a, b, and c given data points (x_i, y_i) using linear least squares?

A: $y = a \cdot 1 + b \cdot x + c \cdot x^2$

B: $y = a^2 \cdot x + b \cdot x + 1 \cdot x$

C: $y = f(a, x) + f(b, x) + f(1, x)$

D: $y = a \cdot f(x) + b \cdot g(x) + c \cdot h(x)$

E: $y = (a \cdot 1 + b \cdot x + c \cdot x)^2$

Write your answer as all the letters for the models that *can* be used with linear least squares, in alphabetical order, without spaces, commas, or other separating characters.

Part 3. Solving least-squares problems

You are given a number of data points (t_i, y_i) in two vectors `t` and `y`.

Set up a matrix A and a right-hand side vector b so that the solution $x = (\alpha, \beta)$ of the least-squares system $Ax \cong b$ is the best fit (in the 2-norm) to $y(t) = \alpha + t\beta$ to the given data.

INPUT: `t` and `y`

OUTPUTS: `A` and `b`

```
import numpy as np

b =
A =
```

Part 4. Solving least-squares problems (II)

This is a continuation of the last problem. This time, you are given the matrix A and the right-hand side vector b, and your goal is to compute the coefficients a and b in the least-squares solution vector $x = (a, b)$ so that $y(t) = a + tb$ is the best fit (in the 2-norm) to the given data.

Also use the function `plot_solution(a, b)` to visualize your result.

Use a QR factorization of A (from `scipy.linalg.qr`) to solve the least-squares problem $Ax \cong b$.

INPUT:

- System matrix `A` and right-hand side vector `b`
- Plotting function `plot_solution(a, b)`

OUTPUTS:

- `alpha`, `beta`

```
import scipy.linalg as la

alpha =
beta =

plot_solution(a, b)
```