Numerical Methods (CS 357)
Worksheet

Part 1. Objectives

- **Long-term goal:** Fitting a model to data using least squares (getting close)
- Solving a least-squares problem using QR
- Setting up coefficient fitting as a least-squares problem

Part 2. Suitable Models for Linear Least-Squares

For which of the following models can you find the coefficients \(a, b,\) and \(c\) given data points \((x_i, y_i)\) using linear least squares?

A: \(y = a \cdot 1 + b \cdot x + c \cdot x^2\)
B: \(y = a^2 \cdot x + b \cdot x + 1 \cdot x\)
C: \(y = f(a, x) + f(b, x) + f(1, x)\)
D: \(y = a \cdot f(x) + b \cdot g(x) + c \cdot h(x)\)
E: \(y = (a \cdot 1 + b \cdot x + c \cdot x)^2\)

Write your answer as all the letters for the models that *can* be used with linear least squares, in alphabetical order, without spaces, commas, or other separating characters.

Part 3. Solving least-squares problems

You are given a number of data points \((t_i, y_i)\) in two vectors \(t\) and \(y\).

Set up a matrix \(A\) and a right-hand side vector \(b\) so that the solution \(x = (\alpha, \beta)\) of the least-squares system \(Ax = b\) is the best fit (in the 2-norm) to \(y(t) = \alpha + t\beta\) to the given data.

**INPUT:** \(t\) and \(y\)
**OUTPUTS:** \(A\) and \(b\)

```
import numpy as np

b = 
A =
```
Part 4. Solving least-squares problems (II)

This is a continuation of the last problem. This time, you are given the matrix $A$ and the right-hand side vector $b$, and your goal is to compute the coefficients $a$ and $b$ in the least-squares solution vector $x = (a, b)$ so that $y(t) = a + tb$ is the best fit (in the 2-norm) to the given data.

Also use the function `plot_solution(a, b)` to visualize your result.

Use a QR factorization of $A$ (from `scipy.linalg.qr`) to solve the least-squares problem $Ax \approx b$.

**INPUT:**

- System matrix $A$ and right-hand side vector $b$
- Plotting function `plot_solution(a, b)`

**OUTPUTS:**

- $alpha, beta$

```python
import scipy.linalg as la

alpha =
beta =

plot_solution(a, b)
```