

Worksheet

Part 1. Linear Algebra review: Rank and Singularity

Let $A \in \mathbb{R}^{n \times n}$ be a matrix.

Consider the following two statements:

A: A is singular.

B: A does not have full rank.

Which of the following is true?

- (A) $A \Leftrightarrow B$
- (B) $A \Leftrightarrow \text{not } B$
- (C) $A \Rightarrow B$
- (D) $A \Leftarrow \text{not } B$
- (E) None of these

Part 2. LU and row/column spaces

Suppose you have a matrix A and its LU factorization $A = LU$. Which of the following is equal to the vector space spanned by the *rows* of A ?

- (A) The space spanned by the columns of L .
- (B) The space spanned by the rows of L .
- (C) The space spanned by the columns of U .
- (D) The space spanned by the rows of U .

Part 3. Rank finding

You are given a square matrix A . You are also given access to the function `m_echelon(A)` (that you just saw in class) which returns an a tuple (M, U) with invertible matrix M and a matrix U in upper echelon form so that $MA = U$.

Compute the rank of A and assign it to `rank`.

INPUT:

- `A`: a matrix as a numpy array
- `m_echelon`: a function to compute the ‘echelon factorization’
- `tol`: the maximum 2-norm at which a row of U is considered zero

OUTPUT:

- `rank`: the rank of the matrix A to tolerance `tol`

```
import numpy as np
import numpy.linalg as la
```