

Numerical Methods (CS 357)

Worksheet

Part 1. Matrix norm approximation

Suppose you know that for a given matrix A three vectors x, y, z as well as a vector norm $\|\cdot\|$, the following are true:

$$\|x\| = 5, \|y\| = 5, \|z\| = 0.5$$

$$\|Ax\| = 20, \|Ay\| = 5, \|Az\| = 5$$

What is the largest lower bound for $\|A\|$ that you can derive from these values?

Part 2. Matrix norm approximation

In the previous question, why was the number you determined only a lower bound?

Part 3. Guesstimating a matrix norm

In the figure below, you see all the vectors that were obtained by taking a lot of random vectors with $\|x\|_\infty = 1$ and multiplying them by a matrix A .

Give an estimate for $\|A\|_\infty$. (Doesn't have to be very accurate.)

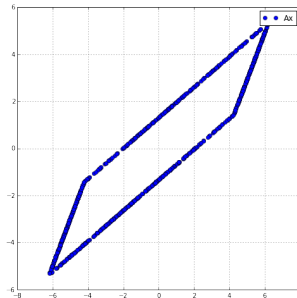


Figure 1: Image of the unit-ball of the infinity norm under A

Part 4. Applying a condition number estimate

Suppose you know that the 2-norm condition number of a matrix A is 20, and that a vector b is known to a relative error (in the 2-norm) of 0.1.

If you solve $Ax = b$ with the approximate b as data, what should the relative error in the solution x at most be?