Numerical Methods (CS 357)

Worksheet

Part 1. Absolute/Relative Error

The average human body temperature is typically quoted as 98.6 degrees Fahrenheit. One could imagine that this was determined by averaging a large number of samples and then rounding to 3 significant digits. If the value of 98.6 is accurate to within plus or minus 0.5 degrees Fahrenheit, what is the maximum relative error? (Round the result to the first signifcant digit.)

Part 2. Absolute Error

Suppose we have the true result

$$x_0 = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}, \qquad \text{and the computed result} \qquad x = \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix},$$

what's the absolute error in the 2-norm? (Two significant digits are enough.)

Part 3. Absolute Error

Suppose we have the true result

$$x_0 = \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix}, \qquad \text{and the computed result} \qquad x = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix},$$

what's the relative error in the 2-norm? (Two significant digits are enough.)

Part 4. Normalizing a vector

Given p and a vector x in $\mathbb{R}^3$, compute a new vector y so that $y = \alpha x$ and $\|y\|_p = 1$.
Hint: Use `numpy.abs()` to find the absolute value of a number.
Hint 2: Calling `numpy.linalg.norm()` is not allowed.
INPUTS:

- `x`, a `numpy` array of shape `(3,)`
- `p`, a p value of a norm

OUTPUTS

- `y`, a multiple of `x` so that $\|x\|_p = 1$

```
import numpy as np
```

Part 5. Norm Criteria

For a vector

$$\vec{x} := \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \qquad \text{is} \qquad \|\vec{x}\| = x^2 + y^2 \qquad \text{a norm?}$$

(A) Yes.

(B) No, because it does not satisfy $\|\alpha\vec{x}\| = |\alpha|\|\vec{x}\|$.

(C) No, because it does not satisfy $\|\vec{x} + \vec{y}\| \leq \|\vec{x}\| + \|\vec{y}\|$.

(D) No, because it does not satisfy $\|\vec{x}\| \geq 0$

(E) No, because it does not satisfy $\|\vec{x}\| = 0 \Leftrightarrow \vec{x} = 0$