Numerical Methods (CS 357)

Worksheet

Part 1. Objectives

- **Long-term goal:** How do I numerically solve an overdetermined linear system? How do I fit a model to data?
- How do I set up a projection matrix?
- How does QR factorization work?
- How does QR factorization relate to Gram-Schmidt orthogonalization?

Part 2. Condition number

Consider the orthogonal matrix

$$Q = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}.$$

What is the 2-norm-based condition number of Q?

Part 3. Projection onto a plane

You are given two vectors `x`, `y` in $\mathbb{R}^3$. Now do the following:

1. Orthonormalize the vectors `x` and `y`, store the result in `xo` and `yo`.
2. Construct the matrix P that orthogonally projects onto the plane spanned by `x` and `y`. Store this matrix in the variable `P`.
3. Project the vector `a` (also given) into the plane and store the result in `projected_a`.
4. Compute the coefficient array `a_coeff` so that:

   $Pa =$ `a_coeff[0]*xo + a_coeff[1]*yo`.

```
import numpy as np
import numpy.linalg as la

xo =
yo =

P =
```

Part 4. Nullspaces and QR factorization

Suppose a matrix A has a nullspace, and also suppose that you have factorization $A = QR$ into an orthogonal matrix Q and an upper triangular matrix R.

Which of the following must be true?

(A) R must have a zero on the diagonal.

(B) Q must have a nullspace.

(C) $Q^T = Q$

(D) $RQ = 0$, i.e. the zero matrix