Lecture 14

WS 13 pb 1

$$e_{n+1} = \frac{|\lambda_2|}{|\lambda_1|} e_n$$

$A - \sigma I$

$$e_{n+1} = \frac{|\lambda_2 - \sigma|}{\underbrace{|\lambda_1 - \sigma|}_{\approx\, 0}} e_n$$

Inverse iteration: A^{-1} with a shift:

$\rightarrow (A - \sigma I)^{-1}$

$$e_{n+1} = \frac{|\lambda_{\text{closest to } \sigma} - \sigma|}{\underbrace{|\lambda_{\text{2nd-closest}} - \sigma|}_{\to \text{small}}}$$

Rayleigh quotient

$$\lambda \approx \frac{x^T A x}{x^T x}$$

Can we fix one-at-a-time?

- Once you know λ_1, x_1:

  similarity transf. $\begin{bmatrix} \lambda_1 & \\ & B \end{bmatrix}$

  restrict to B.

  "Deflation"

- Iterate w/ lots of vectors

Simultaneous iteration

$$X_0 \in \mathbb{R}^{n \times p} \quad (p \leq n)$$

$$X_{k+1} = A X_k$$

Issues:

- normalization / overflow
- X_k increasingly ill-conditioned.

Orthogonal iteration

$$X_0 \in \mathbb{R}^{n \times p} \qquad (p \leq n)$$

$$Q_k R_k = X_k \qquad (\text{reduced})$$

$$X_{k+1} = A Q_k$$

Issues:

- QR fact. n^3
- potentially slow convergence.

$Q_0 R_0 = X_0$

$X_1 = A Q_0$

$Q_1 R_1 = X_1 \quad = A Q_0 \quad \Rightarrow \quad Q_1 R_1 Q_0^T = A$

$X_2 = A Q_1$

$Q_2 R_2 = X_2 \quad = A Q_1 \quad \Rightarrow \quad \left(Q_2 R_2 Q_1^T = A \right)$

$X_0 \in \mathbb{R}^{n \times n}$ has full rank.

$\Downarrow$

once Q's converge: triangular

Schur

$Q^T A Q =$ "upper triang" $\Leftrightarrow Q \quad R_{n+1} Q^T = A$

Sorry about the brief confusion at the end today – all was well in the end, in that the demo matched the fixed version of the identity.

So — nothing to see.