

lecture 14

WS 13 pb 1

$$e_{n+1} = \frac{|\lambda_2|}{|\lambda_1|} e_n$$

$A - \sigma I$

$$e_{n+1} = \frac{|\lambda_2 - \sigma|}{|\lambda_1 - \sigma|} e_n$$

? σ

Inverse iteration: A^{-1} with a shift:

$$\rightarrow (A - \sigma I)^{-1}$$

$$e_{n+1} = \frac{|\lambda_{\text{closest to } \sigma}|}{|\lambda_{\text{2nd-closest}} - \sigma|} e_n$$

\rightarrow small.

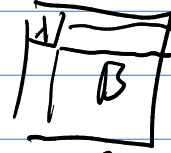
Rayleigh quotient

$$\lambda \approx \frac{x^T A x}{x^T x}$$

Can we fix one-at-a-time?

- Once you know λ_1, x_1 :

similarity transf.



↑
restrict to B.

"Deflation"

- Iterate w/ lots of vectors

Simultaneous iteration

$$X_0 \in \mathbb{R}^{n \times p} \quad (p \leq n)$$

$$X_{k+1} = A X_k$$

Issues:

- normalization / overflow
- X_k increasingly ill-conditioned

Orthogonal iteration

$$X_0 \in \mathbb{R}^{n \times p} \quad (p \leq n)$$

$$Q_k R_k = X_k \quad (\text{reduced})$$

$$X_{k+1} = A Q_k$$

Issues:

- QR fact: n^3
- potentially slow convergence.

$$Q_0 R_0 = X_0$$

$$X_1 = A Q_0$$

$$Q_1 R_1 = X_1$$

$$X_2 = A Q_1$$

$$Q_2 R_2 = X_2$$

$$= A Q_0 \Rightarrow Q_1 R_1 Q_0^T = A$$

$$= A Q_1 \Rightarrow (Q_2 R_2 Q_1^T = A)$$

$X_0 \in \mathbb{R}^{n \times n}$ has full rank

↓

once Q 's converge

$$Q^T A Q = \text{"upper triangular"}$$

↑ triangular

Schur

$$Q R_{n+1} Q^T = A$$

Sorry about the brief confusion at the end
today - all was well in the end, in that the demo
matched the fixed version of the identity.

So - nothing to see. 😊