Office hours moved (today only): 1pm

$$Q_i R_i Q_{i-1}^T = A$$

$$\hat{X}_{i:} = Q_i^T A Q_i$$

$\uparrow$ upper triang.

QR iteration,

"old way" ← orthog. it.

$X_0 = A$

$Q_0 R_0 = X_0$

$\hat{X}_0 = Q_0^T A Q_0$
$= Q_0^T Q_0 R_0 Q_0$
$= R_0 Q_0$

$X_1 = A Q_0 = Q_0 Q_0^T A Q_0$
$= Q_0 \bar{X}_1$
$= Q_0 \bar{Q}_1 \bar{R}_1$

same up to sign

$Q_1 R_1 = X_1$

$\boxed{\begin{array}{l} X_i\text{: iterates of OI} \\ \hat{X}_i = Q_i^T X_i Q_i \\ \bar{X}_i = \text{iterates of QR it.} \end{array}}$

$= \hat{X}_{i-1}$

"new way" ← QR it. (with bars)

$\bar{X}_0 = A$

$\bar{Q}_0 \bar{R}_0 = A$

$\bar{X}_1 = \bar{R}_0 \bar{Q}_0 \quad (= \hat{X}_0)$

$\hat{X}_0 = Q_0^T A Q_0$

$\bar{Q}_1 \bar{R}_1 = \bar{X}_1$

$\bar{X}_2 = \bar{R}_1 \bar{Q}_1 = \bar{Q}_1^T \bar{X}_1 \bar{Q}_1$
$= \bar{Q}_1^T \hat{X}_0 \bar{Q}_1$
$= \bar{Q}_1^T \bar{Q}_0^T A \bar{Q}_0 \bar{Q}_1$
$= Q_1^T A Q_1$
$= \hat{X}_1$

$Q_0 = \bar{Q}_0$
$Q_1 = \bar{Q}_0 \bar{Q}_1$

QR it

$$\left.\begin{array}{l} \bar{X}_0 = A \\ \bar{Q}_i \bar{R}_i = \bar{X}_i \\ \bar{X}_{i+1} = \bar{R}_i \bar{Q}_i \end{array}\right\}$$

check if $\bar{X}$ has converged to upper triag

Drawbacks of orth. / QR it.

- expensive
- slow convergence in $|\lambda_1| \approx |\lambda_2|$

(Todo: defective

Slow convergence!

<u>QR it. w/shift</u>

$\tilde{X}_0 = A$

Choose σ_k

$\tilde{Q}_k \tilde{R}_k = \tilde{X}_k - \sigma_k I$

$\tilde{X}_{k+1} = \tilde{R}_k \tilde{Q}_k + \sigma_k I$

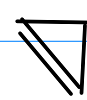
$\tilde{Q}_0 \tilde{R}_0 = A - \sigma I$

$\tilde{X}_1 = \tilde{R}_0 \tilde{Q}_0 + \sigma I$

$= \tilde{Q}_0^T (A - \sigma I) \tilde{Q}_0 + \sigma I$

$= \tilde{Q}_0^T A \tilde{Q}_0 \cancel{- \tilde{Q}_0^T I \tilde{Q}_0 \cdot \sigma + \sigma I}$

$\tilde{Q}_0$ different



<u>expense:</u>

full matrix $\longrightarrow$ upper Hessenberg $\rightarrow$ upper triang

QR using Givens

A defective:

~~just produces more zeros in R~~

Schur form exists, no issue.