Office hours moved (today only): 1 pm

\[ Q_i R_i Q_{i-1}^\dagger = A \]

\[ X_i := Q_i A Q_i \]

upper triangle
QR iteration,

"old way" = orthogonal it.

\[ X_0 = A \]
\[ Q_0 R_0 = x_0 \]
\[ X_0 = Q_0^T A Q_0 \]
\[ = Q_0^T Q_0 L_0 Q_0 \]
\[ = L_0 Q_0 \]

"new way" = QL it. (with bars)

\[ \bar{X}_0 = A \]
\[ Q_0 \bar{R}_0 = A \]
\[ \bar{X}_i = \bar{R}_i \bar{Q}_i \quad (= \bar{x}_0) \]

\[ X_1 = A Q_0 = Q_0 Q_0^T A Q_0 \]
\[ = Q_0 \bar{x}_i \]
\[ = Q_0 \bar{Q}_i \bar{R}_i \]

\[ X_i \text{ iterated} \]
\[ X_i = Q_i^T X_i Q_i \]
\[ X_i \text{ iterated } n \text{ QL it.} \]

\[ Q_k R_k = x_k \]

\[ X_1 = \bar{R}_1 \bar{Q}_1 \]
\[ = \bar{Q}_1^T \bar{x}_1 \bar{Q}_1 \]
\[ = \bar{Q}_1^T \bar{Q}_1 A \bar{Q}_1 \bar{Q}_1 \]
\[ = \bar{Q}_1^T A_k \bar{Q}_1 \bar{Q}_1 \]
\[ = \bar{Q}_1^T A_k \bar{Q}_1 \]

\[ \bar{x}_1 = \bar{X}_1 \]

check if \( \bar{X} \) has converged to upper-triag.
Drawbacks of ortho-QR:

- expensive
- slow convergence in $|x_i x_i^T|$

(ToDo: derivatives)

Slow convergence:

QR it. w/shift

\[
\begin{align*}
\tilde{x}_0 &= A \\
\text{Choose } \sigma_k \\
\tilde{Q}_k \tilde{E}_k &= \tilde{x}_k - \sigma_k I \\
\tilde{x}_{k+1} &= \tilde{Q}_k \tilde{Q}_k + \sigma_k I \\
\tilde{Q}_0 \tilde{R}_0 &= A - \sigma I \\
\tilde{x} &= \tilde{Q}_0 \tilde{Q}_0 + \sigma I = \tilde{Q}_0 (A - \sigma I) \tilde{Q}_0 + \sigma I = \tilde{Q}_0 (A - \sigma I) \tilde{Q}_0 + \sigma I
\end{align*}
\]

\[\tilde{Q}_0 \text{ different}\]

\[
\text{expense:} \quad \nabla
\]

Full matrix $\rightarrow$ upper Hessenberg $\rightarrow$ upper tridiagonal

All using Givens
A defective:

just produces more zeros in \( \mathbb{R} \)

Sanner form exists, no issue.