4.5 Krylov space methods

QR it: $\quad \text{span}\{A^\ell y_1, \ldots, A^\ell y_k\}$

Krylov: $\quad \text{span}\{\underbrace{x}_{x_0}, \underbrace{Ax}_{x_1}, \underbrace{A^2x}_{x_2}, \ldots, \underbrace{A^{k-1}x}_{x_{k-1}}\}$

Define $K_k := [x_0 \; x_1 \; x_2 \ldots x_{k-1}] \leftarrow n \times k$

$$AK_n = [x_1 \; x_2 \qquad x_n]$$

$$= K_n \, [e_2 \, e_3 \, e_4 \ldots e_n \, K^{-1}x_n] \qquad e_2 = (0, 1, 0 \ldots)$$

$$\underbrace{K_n^{-1} A K_n} = \begin{pmatrix} 0 & & \\ & \ddots & \\ & & \end{pmatrix} \leftarrow \text{upper Hessenberg}$$

$$Q_n R_n = K_n \to Q_n = K_n R_n^{-1} \to Q_n^{-1} = R_n K_n^{-1}$$

$$Q_n^T A Q_n = Q_n^{-1} A Q_n$$

$$= R_n \underbrace{K_n^{-1} A K_n}_{\text{Hessenberg}} R_n^{-1}$$

$$= ◹◹ = H$$

Fact:

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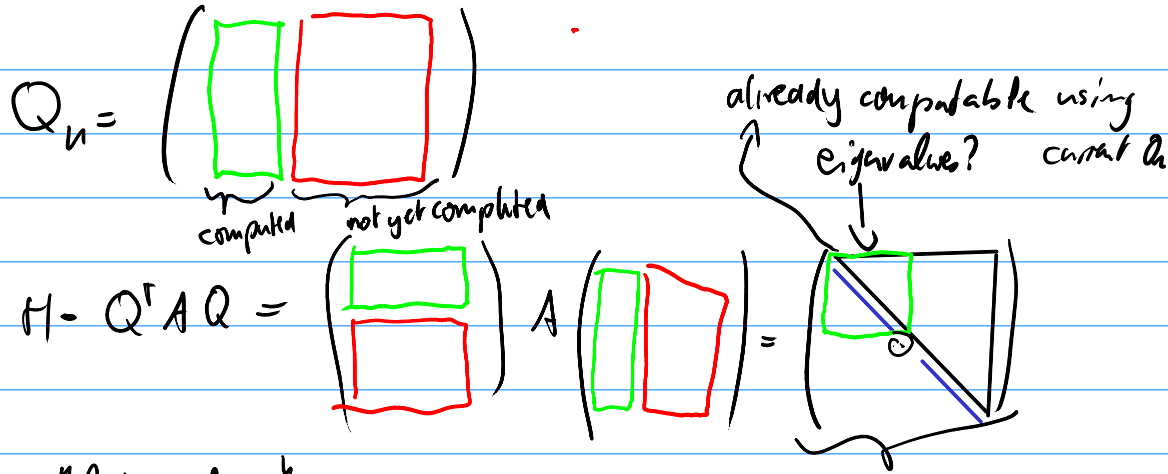
$$AQ_n = Q_n H \xrightarrow[q_i^T]{q\text{th col.}} \quad \underline{\underline{q_i}}^T A q_j = H_{ij}$$



"Ritz values"

expensive? → simply restart w/ new initial vect.

symmetric? → Lanczos iteration

5 Nonlinear equations

Develop machinery $f(\vec{x}) = \vec{0}$ $\quad f: \mathbb{R}^n \to \mathbb{R}^m$

$$f(\vec{x}) = \vec{y}$$

$$\hat{f}(\vec{x}) := f(\vec{x}) - \vec{y}$$

$$\hat{f}(\vec{x}) \stackrel{!}{=} \vec{0}$$

$f_1(\ldots) = 0$

$f_2(\ldots) = 0$