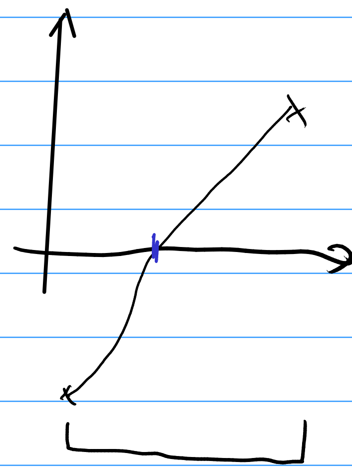
Existence

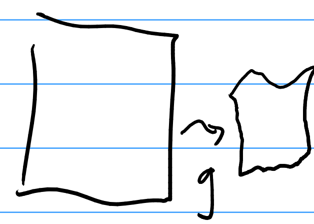
- Intermediate value thm.



"bracket"

1D
nD unhelpful

- Inverse function
- Contraction mapping



$$g : \mathbb{R}^n \to \mathbb{R}^n$$

$$\|g(x) - g(y)\| \leq C \|x - y\|$$

$$0 \leq C < 1$$

Closed set $\to$
$S \subseteq \mathbb{R}^n$ $\Rightarrow$ There exists a unique fixed point of g.
$g : S \to S$

$$g(x^*) = x^*$$

$$0 = f(x) = x - g(x) \qquad f(x^*) = 0$$

Uniqueness?

No.

Sensitivity

$$f(x)=0 \iff f^{-1}(0)=x$$

cond(root finding) = cond(evaluation of inverse)

x is a **Multiple roots** (1D)

$$\left.\begin{aligned} f(x)&=0\\ f'(x)&=0\\ f''(x)&=0\\ &\vdots\\ f^{(m-1)}(x)&=0\\ f^{(m)}(x)&\neq 0 \end{aligned}\right\}$$ "multiplicity m"

Why are multiple roots problematic?

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5.7 Convergence of iterative processes

Let $e_k = \hat{u}_k - u$. (where $\hat{u}_k$ is the kth iterate)

$$\|e_{k+1}\| \leq C \cdot \|e_k\|$$

$C < 1$ for convergence

Faster convergence:

$$\frac{\|e_{k+1}\|}{\|e_k\|} \leq C \qquad \|e_{k+1}\| \leq C \cdot \|e_k\|^r \leq C \cdot \|e_k\|$$

Definition: Iteration has conv. rate r :$\Leftrightarrow$

$$\lim_{k \to \infty} \frac{\|e_{k+1}\|}{\|e_k\|^r} = C > 0$$

Why limit? "allows some bumps"

where error gets bigger.

$r=1 \rightarrow$ linear conv.

$r=2 \rightarrow$ quadratic

$r=3 \rightarrow$ cubic

> 1 ; superlinear

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