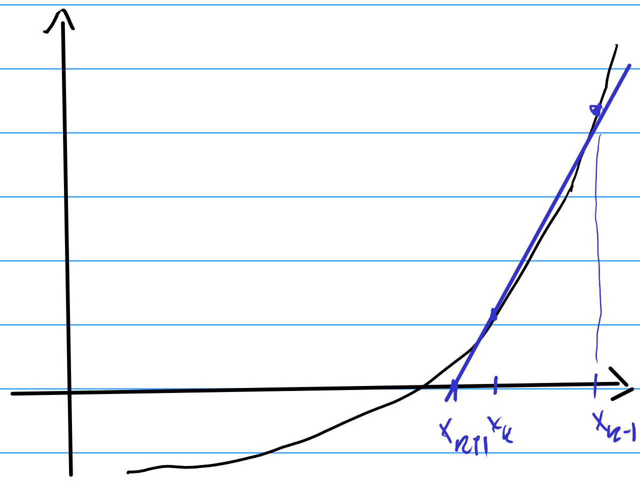
Secant method



$$f'(x_k) \approx \frac{f(x_k)-f(x_{k-1})}{x_k - x_{k-1}}$$

Rate of convergence $(1+\sqrt{5})/2 = 1.618$

Drawbacks: → two starting guesse

→ slower than Newton

→ locally conv.

"trust-region methods"

WS 18 p1

(5.4) Methods in n dimensions

("Systems of eqns")

Fixed point iteration

$x_0 = \langle \text{starting guess} \rangle$

$x_{k+1} = g(x_k)$

1D

Converges locally $\rho(J_g(x^*)) < 1$ ("$|f'(x^*)| < 1$")

$$g'(x) = J_g(x) = \begin{pmatrix} \partial_{x_1} g_1 & \cdots & \partial_{x_n} g_1 \\ \vdots & & \vdots \\ \partial_{x_1} g_n & \cdots & \partial_{x_n} g_n \end{pmatrix}$$

Converges linearly

Newton

$$f(x+s) \approx f(x) + J_f(x)s \stackrel{!}{=} 0$$

$$\underbrace{J_f(x)}_{\text{matrix}} s = -f(x)$$

$$s = J_f^{-1}(x)(-f(x))$$

$x_0 = \langle \text{starting guess} \rangle$

$$x_{k+1} = x_k + s = x_k + J_f^{-1}(x)(-f(x_k))$$

locally quadratic convergence

Drawbacks:
- → derivative available?
- → solve lin. system every iteration.
- → locally convergent.

Secant-ish methods

$$(*) \quad \tilde{J}(x_{k+1} - x_k) \approx f(\underset{x_k+s}{x_{k+1}}) - f(x_k)$$

n^2 unknowns

n equations

How many eqns?
How many unknowns?

Broyden's method

- satisfies (*) secant condition
- makes the least possible change to $\tilde{J}$

⑥ Optimization

$f(x) \overset{!}{=} 0$

$\min_x \|f(x)\|$ { don't really need the norm
→ build it into f.

$f: \mathbb{R}^n \to \mathbb{R}$ Objective Function

$$\min_x f(x) \quad \text{subject to} \quad \overbrace{g(x) = 0}^{\text{feasible points}}, \; h(x) \le 0 \quad \text{constraints}$$

↳ "constrained optimization"

linear / nonlinear programming

6.2 Existence / uniqueness

continuous

- $f: S \to \mathbb{R}$ $\quad S \subseteq \mathbb{R}^n$ closed and bounded

  $\Rightarrow$ has minimum

- $f: S \to \mathbb{R}$ $\quad S \subseteq \mathbb{R}^n$ unbounded

  f coercive

  if $\lim_{\|x\| \to \infty} f(x) = +\infty$

  $\Rightarrow$ f has a global minimum