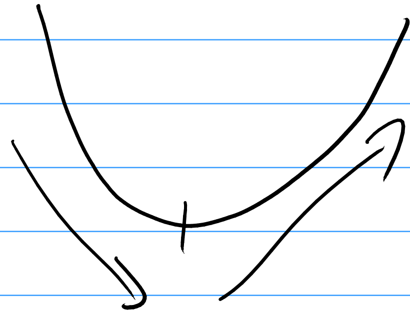
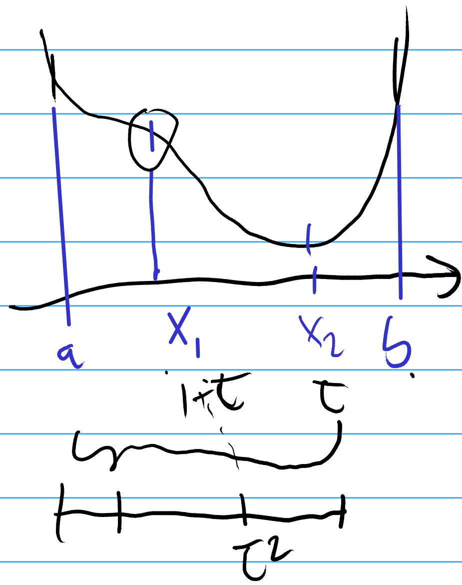
Golden Section Search



unimodal



- symmetric

$$x_1 = a + \tau(b-a)$$
$$x_2 = a + (1-\tau)(b-a)$$

- Best value of τ

guarantees reuse of function values

$$\tau^2 = 1-\tau$$
$$\tau = \frac{\sqrt{5}-1}{2} = .618$$
$$\frac{1+\sqrt{5}}{2} = \text{"golden ratio"}$$

converges linearly

Newton's method

$$f(x+h) \approx f(x) + f'(x)h + f''(x)\cdot\frac{h^2}{2} = \hat{f}(h)$$

$$\hat{f}'(h) = f'(x) + f''(x)\cdot h = 0$$

$$h = -\frac{f'(x)}{f''(x)}$$

$$\left[\begin{array}{l} x_0 = \langle \text{starting guess} \rangle \\ x_{k+1} = x_k - \frac{f'(x)}{f''(x)} \end{array}\right] \quad \text{solves } f'(x) = 0 \text{ using Newton's method}$$

quadratic convergence

Idea: Combine slow-and-safe with fast'n'risky