⚠ office hours moved to 11am

6.6 Methods in n D

$$\min_x f(x)$$

Steepest descent

$x_0 = \langle \text{starting guess} \rangle$

$s_k = -\nabla f(x_k)$

Choose α s.t. $f(x_k + \alpha s_k)$ is minimized.

$x_{k+1} = x_k + \underline{\alpha} \cdot s_k$

WS21 p1

Newton

$$f(x+s) = \underbrace{f(x) + \nabla f(x)^T s + \frac{1}{2} s^T H_f(x) s}_{\text{quadr. approx.}\ \hat{f}(s)} + O(s^3)$$

$\nabla \hat{f}(s) = 0$

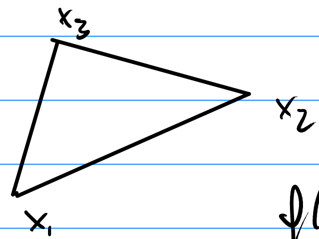
$\rightsquigarrow H_f(x)\, s = -\nabla f(x)$ $\rightsquigarrow s = -H_f^{-1}(x) \nabla f(x)$

$x_0 = \langle \text{starting guess} \rangle$

$x_{k+1} = x_k - H_f^{-1}(x) \nabla f(x)$

$$x^T x \to x^T H_f x$$

Nelder-Mead "crawling amoeba"



$$f(x_1) < \dots < f(x_3)$$

6.7 Nonlinear least squares

$$f(x) = \|r(x)\|_2^2$$

$$r(x) = y - f(x)$$

$$\phi(x) = \frac{1}{2} r(x)^T r(x)$$

$$\frac{\partial}{\partial x_i} \phi = \frac{\partial}{\partial x_i} \left(\frac{1}{2} \sum_{j=1}^{n} r_j^2(x) \right)$$

$$= \sum_{j=1}^{n} r_j(x) \cdot \underbrace{\frac{\partial}{\partial x_i} r_j(x)}_{J_r(x)}$$

$$= J_r^T(x) \cdot r(x)$$