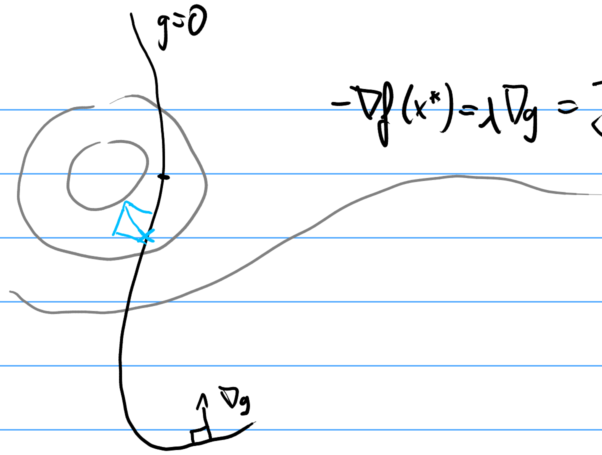



$$-\nabla f(x^*) = \lambda \nabla g = J_g^T \tilde{\lambda}$$

vector

## Lagrangian fuction

$$\mathcal{L}(x,\lambda) = f(x) + \tilde{\lambda}^T g(x)$$

$$\nabla \mathcal{L}(x,\lambda) = \begin{pmatrix} \nabla_x \mathcal{L} \\ \nabla_\lambda \mathcal{L} \end{pmatrix} = \begin{pmatrix} \nabla f + \lambda^T \nabla g \\ g \end{pmatrix} \overset{!}{=} 0$$

$\nabla \mathcal{L} = 0 \leftarrow$ to solve, use Newton

## Sequential quadratic programming

## Inequality constraints

$$\min_x f(x) \quad \text{subject to} \quad g(x)=0 \qquad h(x)\le 0$$
$$g:\mathbb{R}^n \to \mathbb{R}^m \qquad h:\mathbb{R}^n \to \mathbb{R}^p$$

Idea: • Construct $\mathcal{L}$ as before


$$\mathcal{L}(x,\vec{\lambda}_1,\vec{\lambda}_2) = f(x) + \lambda_1^T g + \lambda_2^T h$$

← on the bdy: constraint "active" / "binding"

• If $h_i$ is inactive at $x^*$, force $\lambda_{2,i}=0$

$h_i(x)<0$

$h_i(x)\cdot\lambda_{2,i}=0$ ("complementarity condition")

necessary

⊗ $\nabla_x \mathcal{L}(x^*,\lambda_1^*,\lambda_2^*)=0$

⊗ $g(x^*)=0$

$h(x^*)\le 0$ $\quad \lambda_2\ge 0$

⊗ $h_i\cdot\lambda_{2,i}=0$ $\quad (i=1,\dots,p)$

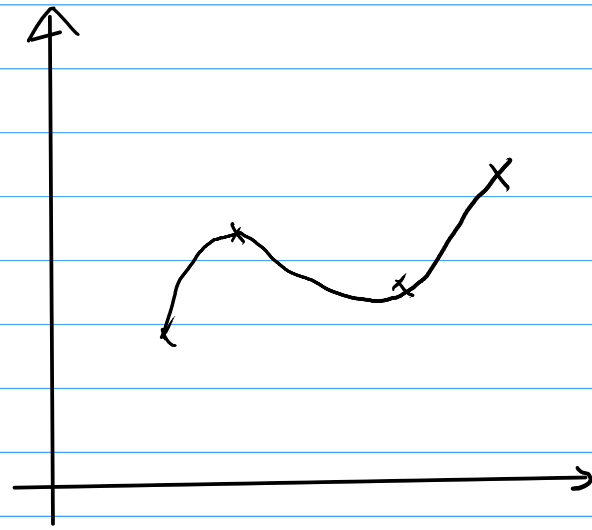
"KKT"

Karush Kuhn Tucker

WS23p1

Feed ⊗ to Newton

## 7 Interpolation



| $x$ | $y$ |
|---|---|
| $x_1$ | $y_1$ |
| $\vdots$ | $\vdots$ |
| $x_m$ | $y_m$ |

**Seeking:** $\alpha_1, \dots, \alpha_n$

$$f(x) = \sum_{j=1}^{m} \alpha_j \varphi_j(x)$$

$$y_i \overset{!}{=} f(x_i) = \sum_{j=1}^{n} \underbrace{\varphi_j(x_i)}_{V_{ij}} \alpha_j = (V\vec{\alpha})_i$$

generalized Vandermonde matrix

"basis matrix"

**Interpolation** $V\vec{\alpha} = \vec{y}$

$$V \begin{pmatrix} \text{basis} \\ \text{coeffs} \end{pmatrix} = \begin{pmatrix} \text{values} \\ \text{at} \\ \text{nodes} \end{pmatrix}$$

$$V^{-1} \begin{pmatrix} \text{values} \\ \text{at} \\ \text{nodes} \end{pmatrix} = \begin{pmatrix} \text{basis} \\ \text{coeffs} \end{pmatrix}$$

Why? $\Rightarrow$ Give me a function to work with

- Evaluate
- Differentiate

$$f'(x) = \sum_{j=1}^{n} \alpha_j \varphi_j'(x)$$

$$\underbrace{V_{\varphi'} \, V_\varphi^{-1} \vec{y}}$$