Office hours at 11am today

7 Interpolation

Existence $\vec{y} \in$ column span V

Uniqueness $N(V) = \{0\}$

Conditioning $\text{cond}(V)$

Choices:
- basis?
- nodes?

7.1 Nodes and bases

Monomial $1, x, x^2, x^3$

equispaced

Lagrange Basis

$$V \stackrel{!}{=} I$$

$$\varphi_j(x_i) = \begin{cases} 1 & i=j \\ 0 & \text{otherwise} \end{cases}$$

nodes x_1, x_2, x_3 $\quad \varphi_1(x_1)=1 \quad \varphi_1(x_2)=0 \quad \varphi_1(x_3)=0$

$$\varphi_1(x) = \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)}$$

← enforce zeros

← make it 1 at the node

$$\varphi_2(x) = \frac{(x-x_1)}{(x_2-x_1)} \cdot \frac{(x-x_3)}{(x_2-x_3)}$$

$$\varphi_j(x) = \frac{\prod_{k=1, k\neq j}^{m} (x-x_k)}{\prod_{k=1, k\neq j}^{m} (x_j-x_k)}$$

Lagrange polynomial

Downsides: • expensive

• hard to work with

WS24p1

Changed

Newton $\varphi_j(x) = \prod_{k=1}^{j-1} (x - x_k)$

$V = ? \rightarrow V_{ij} = \varphi_j(x_i) = 0$ if $i < j$

$\rightarrow$ triangular

"divided differences" to compute coefficients

Orthogonal polynomials

Define "dot" / inner product:

$$\langle f, g \rangle = \int_a^b f(x)\, g(x)\, w(x)\, dx$$

"weight" function

> 0

for now $= 1$

$$\|f\| = \sqrt{\langle f, f \rangle} \leftarrow \text{"}L^2\text{"}$$

ℓ^2

Gram-Schmidt on $1, x, x^2, x^3, \ldots \rightarrow$ "Legendre polynomials

Three-term recurrence $P_k \leftarrow$ kth Legendre-poly

$$(k+1)\, P_{k+1}(x) = (2k+1)\, x\, P_k(x) - k\, P_{k-1}(x)$$

$\rightarrow$ DLMF