Hermite: $(-\infty, \infty)$ $w(x) = e^{-x^2}$

$$(f, g) = \int_{-\infty}^{\infty} f(x) g(x) e^{-x^2} dx$$

<u>Chebyshev:</u>

$$T_k(x) = \cos(k \cos^{-1}(x))$$

- actually polynomial
- $-1 \leq T_k([-1, 1]) \leq 1$
- roots generate nodes

WS25p1

7.2 Error result:

Assume $x_1 < x_2 < \cdots < x_n$.

$$f(x) - p_{n-1}(x) = \frac{f^{(n)}(\xi)}{n!} \underbrace{(x-x_1)(x-x_2)\cdots(x-x_n)}_{\otimes}$$

Suppose $|f^{(n)}(x)| \le M$ on $[x_1, x_n]$:

$$\max_{x \in [x_1, x_n]} |f(x) - p_{n-1}(x)| \le \frac{M}{4n} h^n$$

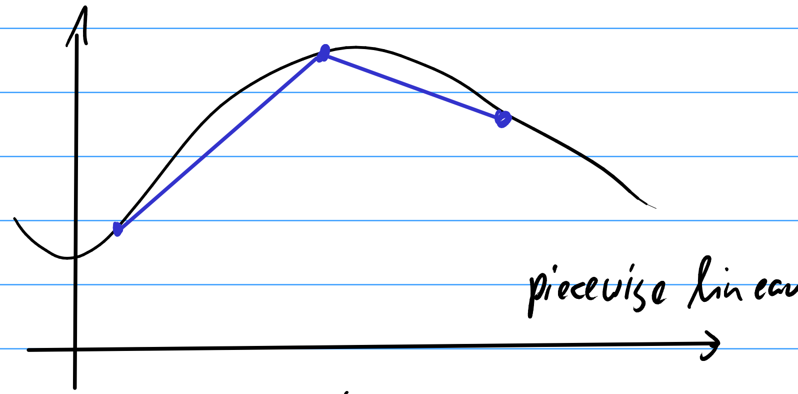
$h = \max_i |x_{i+1} - x_i|$

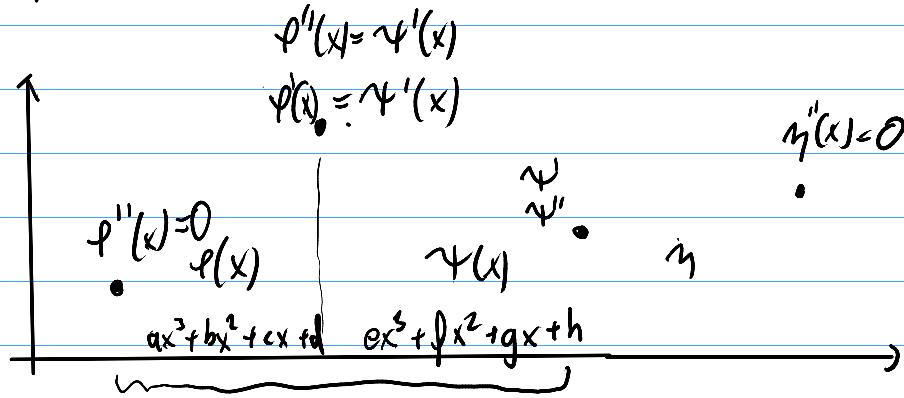
$\otimes$ makes error zero at nodes

Runge phenomenon comes from increasing n-th derivatives

Clustering nodes to edges of interval controls $\otimes$

7.3 Piecewise interpolation





8 variables

4 equations

cubic spline

w/ $\varphi''(x) = 0$ "natural spline"