

Hermite: $(-\infty, \infty)$ $w(x) = e^{-x^2}$

$$(f, g) = \int_{-\infty}^{\infty} f(x)g(x)e^{-x^2} dx$$

Chebyshev:

$$T_n(x) = \cos(n \cos^{-1}(x))$$

- actually polynomial

- $-1 \leq T_n(-1, 1) \leq 1$

- roots generate nodes

WS25 p1

7.7 Error result:

Assume $x_1 < x_2 < \dots < x_n$.

$$f(x) - p_{n-1}(x) = \frac{f^{(n)}(\xi)}{n!} (x-x_1)(x-x_2) \dots (x-x_n)$$

(⊗)

Suppose $|f^{(n)}(x)| \leq M$ on $[x_1, x_n]$:

$$\max_{x \in [x_1, x_n]} |f(x) - p_{n-1}(x)| \leq \frac{M}{n!} h^n$$

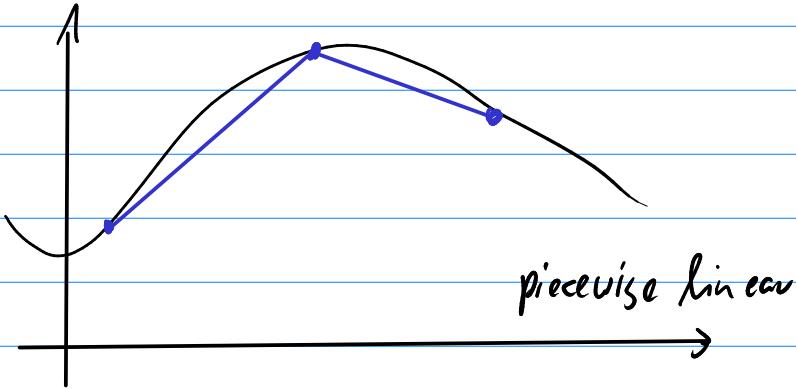
$$h = \max_i |x_{i+1} - x_i|$$

(⊗) makes error zero at nodes

Runge phenomenon comes from increasing derivatives
 $\sqrt[n]{\text{ }}$

Clustering nodes to edges of interval controls (⊗)

7.3 Piecewise interpolation



A graph of a cubic function $y(x) = ax^3 + bx^2 + cx + d$. It shows the function y , its first derivative y' , and its second derivative y'' . The second derivative is zero at the endpoints. The function is approximated by a cubic spline.

8 variables

4 equations

cubic spline

w/ $y''(x)=0$ "natural spline"