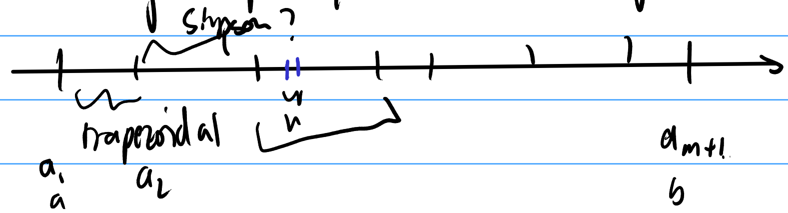
⚠ Office hours moved to 3pm

Reasons to hate Newton-Cotes:

- Method of undet. coeff. increasingly ill-cond with n.
- Wiggly weights → quad. unstable for high n
- all the fun of high-order interp.
- hard to extend to high number of points.

8.3 Composite quadrature

<u>Idea:</u> string a few quadrature rules together



<u>Error:</u> (recall) single interval $\left|\int f - p_n\right| \le C\cdot h^n \|f^{(n)}\|_\infty$

$$\left|\int_a^b f(x)\,dx - \sum_{j=1}^{m}\sum_{i=1}^{n} w_{ji} f(x_{ji})\right|$$

$$\le C\cdot\|f^{(n)}\|_\infty \cdot \sum_{j=1}^{m}\left(\frac{a_{j+1}-a_j}{n}\right)^n$$

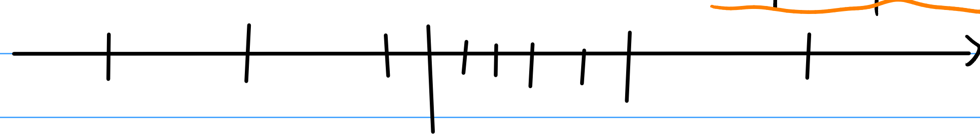
$$\le C\,\|f^{(n)}\|_\infty \underbrace{\sum_{j=1}^{m}(a_{j+1}-a_j)}_{(b-a)}\ \underbrace{\frac{1}{n}\left(\frac{1}{m\cdot n}\right)^{n-1}}_{\text{doesn't depend on } j}$$

$$\le C\cdot\|f^{(n)}\|_\infty\,(b-a)\cdot\underbrace{\frac{1}{n}}_{\text{meh.}}\,h^{n-1}$$

→ Composite quadrature loses an order compared to underlying rule.

<u>Recall:</u> Reducing interval size reduces the error

"adaptive quadrature"



8.4 Gaussian quadrature

So far: nodes prescribed

Now: nodes from quadrature

$$\frac{1}{k+1}\left(b^{k+1} - a^{k+1}\right) = \sum \omega_i \, x_i^k$$

try: solve for weights and nodes

→ nonlinear

→ too hard

$p(x)$: poly of degree n with

$$\int_a^b p(x)\, x^k \, dx = 0 \qquad k = 0, \dots, n-1$$

Then:

- p has n simple, real roots
- using these roots as nodes as quad. nodes yields an interpolatory quad. that is exact up to degree $2n-1$.

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