Exam on Wed.

5-8

(9.2) Numerical Methods (pt. I)

$$y'(t) = f(y(t))$$

$$y(t_0) = y_0$$

h : $t_1 = t_0 + h$, $t_2 = t_0 + 2h$ ...

$\uparrow$ y_1 $\uparrow$ y_2 ...

"Integral equation"

$$y(t) = y_0 + \int_{t_0}^{t} f(y(\tau))\, d\tau$$

Idea: use trapezoidal rule:

$$y_{k+1} = y_k + f(y_k)\cdot\frac{h}{2} + f(\underbrace{y_{k+1}}_{?})\cdot\frac{h}{2}$$

have to solve an eqn

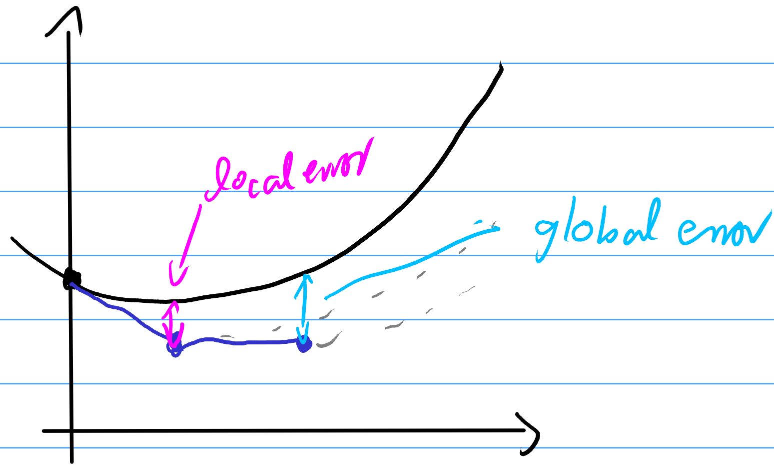
("backward Euler")

Idea 2: use left-handed rectangle rule

$$y_{k+1} = y_k + h \cdot f(y_k)$$

("Euler / forward Euler")

9.3 Accuracy and Stability



local error: $\ell_k = y_{k+1} - u_k(t_{k+1})$ $u_\ell(t)$ solves ODE with IC: $u(t_\ell) = y_\ell$

global error: $y_k - y(t_k)$

global error $\neq \sum$ (local errors)
$>$

Time integrator is of order p if $\ell_k = O(h^{p+1})$

$\sum$ local errors over unit-length

$\frac{1}{h} \cdot O(h^{p+1}) = O(h^p)$

#steps in $(0,1)$

Stability

Instability:

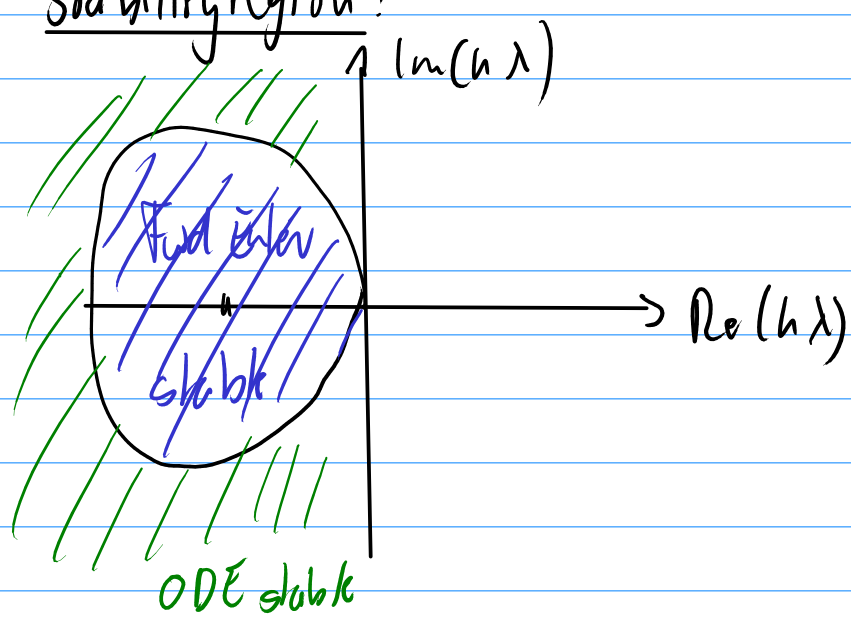
- could be ODE instability
- could be the method

Fw Euler for: $y'(t) = \lambda y(t)$

$$y_{k+1} = y_k + h\lambda y_k$$
$$= y_k(1+h\lambda)$$
$$|y_{k+1}| = |y_k| \underbrace{|1+h\lambda|}_{\text{growth factor}} = |y_0| \cdot |1+h\lambda|^{k+1}$$

$(|h\lambda - (-1)|)$ stable $\iff |1+h\lambda| \le 1$ radius

"Stability region":



Stability in the nonlinear case

Fw Euler: (A) $y_{k+1} = y_k + h f(y_k)$

Taylor: (B) $y(t_{k+1}) = y(t_k) + h f(y(t_k)) + O(h^2)$

(A)-(B) $e_{k+1} = y_{k+1} - y(t_{k+1}) = \underbrace{\underbrace{y_k - y(t_k)}_{e_k} + h\left(f(y_k) - f(y(t_k))\right)}_{\text{propagated error}} + \underbrace{O(h^2)}_{\substack{l_k \\ \text{local error}}}$