Fw Euler stable for $y' = \lambda y$ iff

$$\begin{aligned} y_{k+1} &= y_k + h\lambda y_k \\ &= y_k\underline{(1+h\lambda)} \end{aligned}$$

<u>Stability in the nonlinear case</u>

Fw Euler $\quad y_{k+1} = y_k + h\cdot f(y_k)$

Taylor: $\quad y(t_{k+1}) = y(t_k) + h\cdot f(y(t_k)) + O(h^2)$

$$e_{k+1} = y_{k+1} - y(t_{k+1}) = \underbrace{\underbrace{y_k - y(t_k)}_{e_k} + h\big(f(y_k) - f(y(t_k))\big)}_{\text{propagated error}} + \underbrace{O(h^2)}_{\substack{l_k \\ \text{local error}}}$$

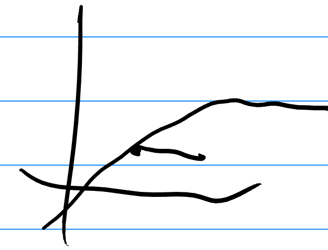
$$\bar{J}_f = \int_0^1 J_f\big(\alpha y_k + (1-\alpha)\, y(t_k)\big)\, d\alpha$$

$$\begin{aligned} e_{k+1} &= e_k + h\bar{J}_f e_k + O(h^2) \\ &= e_k(I + h\bar{J}_f) + O(h^2) \end{aligned}$$

9.4 Numerical Methods (pt. II)

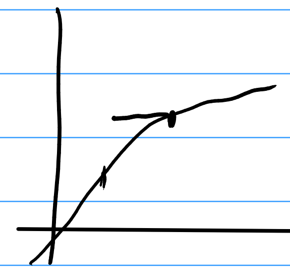
$$y(t_{k+1}) = y(t_k) + \int_{t_k}^{t_{k+1}} f(y(\tau))\,d\tau$$

$$\underset{\text{unknown}}{y_{k+1}} = \underset{\text{known}}{y_k} + h \cdot \underset{\text{unknown}}{f(y_{k+1})}$$



Backward Euler

next value det. by formula
→ explicit method



next value det. by solving an equation
→ implicit method

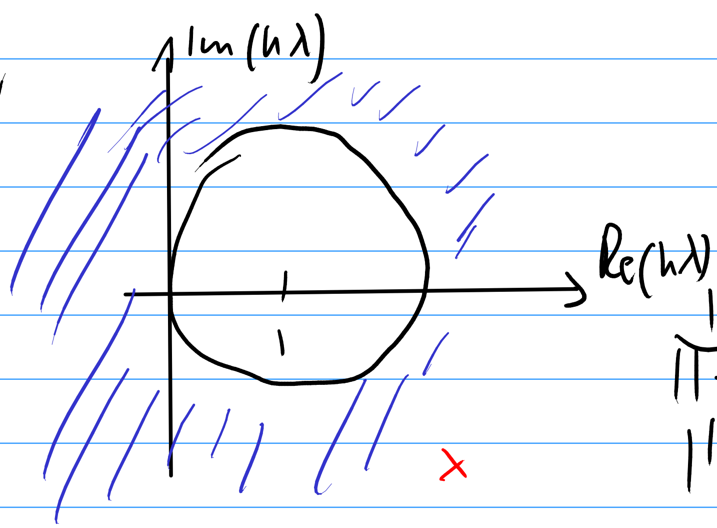
Example: $y'(t) = \lambda y(t)$

$$y_{k+1} = y_k + h\lambda y_{k+1} \rightsquigarrow (1-h\lambda)\, y_{k+1} = y_k$$

$$y_{k+1} = \underbrace{\frac{1}{1-h\lambda}}\, y_k$$

stable iff $|\cdot| \leq 1$

Bw Euler



$$|1-h\lambda| \leq 1$$

$$|1-h\lambda| \geq 1 \iff |(h\lambda) - 1| \geq 1$$

Fv Euler

x