tableau X

Runge-Kutta $\quad y'(t) = f(t,y)$

Butcher tableau

$y_k + h(\qquad)$

$$r_1 = f(t_k + c_1 h,\ a_{11}\cdot r_1 + \dots + a_{1s}\cdot r_s)$$

"stages" $\quad \vdots$

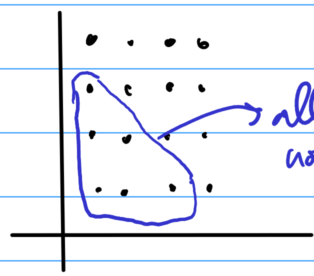
$$r_s = f(t_k + c_s h,\ a_{s1}\cdot r_1 + \dots + a_{ss}\cdot r_s)$$

$$\begin{array}{c|ccc}
c_1 & a_{11} & \cdots & a_{1s} \\
\vdots & & & \\
c_s & a_{s1} & & a_{ss} \\
\hline
 & b_1 & \cdots & b_s
\end{array}$$

$s = \#$ stages

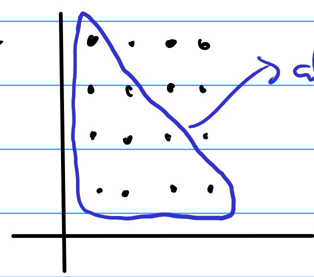
$$y_{k+1} = y_k + h(b_1\cdot r_1 + \dots + b_s\cdot r_s)$$

Explicit:



allowed to be non-zero

Diagonally implicit



allowed to be non-zero

"Embedded" Runge-Kutta method

$$y_{k+1} = y_k + h(b_1\cdot r_1 + \dots b_s r_s)$$

$$\begin{array}{c|ccc}
\\
\hline
 & b_1 & \cdots & b_s \\
 & \tilde{b}_1 & \cdots & \tilde{b}_s
\end{array}$$

$$\tilde{y}_{k+1} = y_k + h(\tilde{b}_1\cdot r_1 + \dots \tilde{b}_s r_s)$$

$\rightsquigarrow$ can use $\tilde{y}_{k+1} - y_{k+1}$ for error estimation

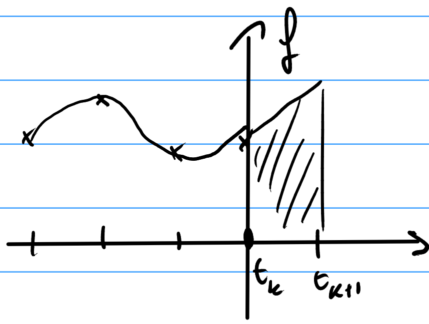
Single-stage, multi-step

$$y_{k+1} = \sum_{i=1}^{M} \alpha_i \, y_{k+1-i} + h \cdot \sum_{i=1}^{N} \beta_i \, f(y_{k+1-i})$$

Known as:

"extrapolation methods"

- If $M=1$, Adams-Bashforth
- If $N=1$, BDFs (backward differencing formulas)



$$y_{k+1} = y_k + \int_{t_k}^{t_{k+1}} f(y(t))\,dt$$

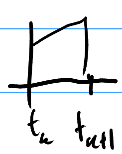
- Implicit varieties also exist.
- Downside: not self-starting.

$$\begin{array}{c|cc} 0 & & \\ 1 & 1 & \\ \hline & \frac{1}{2} & \frac{1}{2} \end{array}$$

Heun's method: (recap)

$$\hat{y}_{k+1} = y_k + h \cdot f(y_k)$$

same



$$y_{k+1} = y_k + \frac{h}{2}\left(f(y_k) + f(\hat{y}_{k+1})\right)$$

$$r_1 = f(y_k + h(\ldots))$$

0

$$r_2 = f(y_k + h \cdot r_1)$$