Problem 1: Elimination matrices
Consider the elimination matrices
\[ A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & -4 & 1 \\ -4 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 \\ 1 & 4 \\ -2 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 1 \\ 1 & 5 \end{pmatrix}. \]

(a) Give an example of a vector \( x \) for which the third and fourth entry of \( Ax \) are zero.

(b) Compute \( AB \) and \( AC \).

Problem 2: LU decomposition
Write down the first column of the factor \( L \) of the (un-pivoted) LU decomposition of
\[ A = \begin{pmatrix} 3 & 4 & 5 \\ 2 & 3 & 1 \\ 9 & 2 & 7 \end{pmatrix}. \]

Problem 3: Conditioning vs Pivoting
What is the relationship between pivoting and conditioning?