Objectives: (1) Understand costs of linear algebra operations, (2) set up least squares problems, (3) understand geometric intuition behind least squares, (3) be familiar with the normal equations.

Problem 1: Set up linear least squares
For each day since the semester started, you’ve written down

- how grumpy you were on day $i$ of the semester, as a value $g_i$ on a scale of 1 to 10,
- the average temperature outside on day number $i$, $t_i$, in degrees Fahrenheit.

(a) You conjecture that a linear relationship exists: $g_i \approx \alpha t_i + \beta$.
Set up a least squares problem to find an estimate of $\alpha$ and $\beta$, written in matrix form $Ax \approx b$. What are $A$, $x$, and $b$?

(b) What would $A$, $x$, and $b$ be for a quadratic relationship $g_i \approx \alpha t_i^2 + \beta t_i + \gamma$?

(c) You come to the conclusion that neither linear nor quadratic relationships are strong enough, and that a higher (but unknown) power of $t_i$ is needed to describe your grumpiness: $g_i \approx \alpha t_i^\beta$.
Take the log of the equations and write the problem in matrix form $Ax \approx b$. What are $A$, $x$, and $b$?

Problem 2: Derive the normal equations
Consider $\varphi(x) = \|r(x)\|^2_2 = \|Ax - b\|^2$. Expand $\varphi$ and take the gradient in $x$ to derive the normal equations.