Numerical Analysis (CS 450)

Worksheet 8

Objectives: **(1)** understand the geometric intuition behind least-squares problems, **(2)** apply the normal equations method and know its drawbacks,

Problem 1: Derive the normal equations

Consider $\varphi(x) = \|r(x)\|_2^2 = \|Ax - b\|^2$. Expand φ and take the gradient in x to derive the normal equations.

Problem 2: Properties of linear least squares problems

	Answer
What is more problematic in an $m \times n$ least squares system with $m > n$: if the rows of A are linearly dependent or the columns?	☐ Rows ☐ Columns
When solving the linear least squares problem $Ax \cong b$, the residual $r = 0$ if and only if $b \notin \operatorname{span}(A)$.	☐ True ☐ False
When solving the linear least squares problem $Ax \cong b$, if the residual $r = 0$, then the solution is unique.	☐ True ☐ False

Problem 3: Usage of QR

(a) If A is an $m \times n$ matrix with $m > n$ and $A = QR$ is a QR factorization, what are the shapes of Q and R?

(b) What are the shapes of Q and R in the *reduced* QR factorization?

(c) Suppose now that $n = m$ (i.e. A is square), and that you have a QR decomposition $A = QR$ of A. Write an algorithm to solve $Ax = b$. You may call the function `back_substitute(M, b)`, where `M` is upper triangular.