Numerical Analysis (CS 450)

Worksheet 9

Objectives: **(1)** Understand how the QR decomposition helps solve linear least squares problems, **(2)** Use Gram-Schmidt and variants to compute a QR decomposition, **(3)** Use the Householder method to compute QR, **(4)** Use the Givens method to compute QR.

Problem 1: Usage of QR

(a) If A is an $m \times n$ matrix with $m > n$ and $A = QR$ is a QR factorization, what are the shapes of Q and R?

(b) What are the shapes of Q and R in the *reduced* QR factorization?

(c) Suppose now that $n = m$ (i.e. A is square), and that you have a QR decomposition $A = QR$ of A. Write an algorithm to solve $Ax = b$. You may call the function `back_substitute(M, b)`, where `M` is upper triangular.

Problem 2: Methods for computing QR

(a) Consider the one-column matrix $A = (a_1, a_2, a_3)^T \in \mathbb{R}^{3\times 1}$. Construct the reduced QR decomposition of A using Gram-Schmidt.

(b) Based on your experience from (a), do you expect the reduced QR factorization of A unique? Why/why not?

(c) Give a Householder transformation H which turns $x = (2, 1, 2)^T$ into a vector of the form $Hx = (*, 0, 0)^T$. (A matrix expression suffices—no need to compute the full matrix.)