Numerical Analysis (CS 450)

# Worksheet 10

**Objectives:** **(1)** Use the Householder method to compute QR **(2)** Use the Givens method to compute QR **(3)** Know properties and applications of the SVD

## Problem 1: Methods for computing QR

(a) What condition must the vector $\mathbf{w}$ satisfy for $\mathbf{I} - \mathbf{w}\mathbf{w}^T$ to be orthonormal?

(b) What situation will you encounter if you apply the QR factorization to a singular matrix?

(c) Suppose you want to solve the least squares problem $\mathbf{A}\mathbf{x} \cong \mathbf{b}$ and you are give an orthonormal $\mathbf{Q}$ matrix so that $\mathbf{Q}\mathbf{A}$ is diagonal. How would you solve $\mathbf{A}\mathbf{x} \cong \mathbf{b}$ in this situation?

## Problem 2: Properties of the SVD

(a) If $\mathbf{A}$ is $m \times n$, what are the dimensions of the matrices in $\mathbf{A} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T$ and what properties do each have?

(b) What assumptions do we need to make about the matrix $A$ to make sure that its SVD exists?

(c) How do the columns of $U$ relate to $A$? How about the columns of $V$? How about each $\sigma_i$?

(d) Assume $\sigma_i \neq 0$ for all $i$ and all matrices to be of size $n \times n$. Write the formula

$$\mathbf{x} = \sum_{i=1}^{n} \mathbf{v}_i \frac{\mathbf{u}_i^T \mathbf{b}}{\sigma_i}$$

using the matrices $\mathbf{U}$ and $\mathbf{V}$, where $\mathbf{u}_i$ are the columns of $\mathbf{U}$, and $\mathbf{v}_i$ are the columns of $\mathbf{V}$.