Numerical Analysis (CS 450)

Worksheet 11

Objectives: **(1)** Know properties and applications of the SVD **(2)** Understand conditioning of eigenvalue problems **(3)** Know effect of matrix transformations on eigenvalues

Problem 1: Properties of the SVD

(a) If $\mathbf{A}$ is $m \times n$, what are the dimensions of the matrices in $\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T$ and what properties do each have?

(b) What assumptions do we need to make about the matrix A to make sure that its SVD exists?

(c) How do the columns of U relate to A? How about the columns of V?

(d) Assume $\sigma_i \neq 0$ for all i and all matrices to be of size $n \times n$. Write the formula

$$\mathbf{x} = \sum_{i=1}^{n} \mathbf{v}_i \frac{\mathbf{u}_i^T \mathbf{b}}{\sigma_i}$$

using the matrices $\mathbf{U}$, $\mathbf{\Sigma}$ and $\mathbf{V}$, where $\mathbf{u}_i$ are the columns of $\mathbf{U}$, and $\mathbf{v}_i$ are the columns of $\mathbf{V}$.

Problem 2: Eigenvalue problem: properties and conditioning

(a) A 3×3 matrix has 2 distinct eigenvalues. Is it necessarily defective?

(b) Why is the eigenvalue problem well-conditioned for symmetric matrices?

(c) Suppose $\lambda = 2$ is an eigenvalue of A. Name an eigenvalue of $(A^2 - 2)^{-1}$.