Objectives: (1) Use $n$-dimension Taylor approximation to derive Newton’s method (2) Understand limitations of steepest-descent methods

Problem 1: Quadratic approximation and Newton

(a) Write down the $O(h^3)$ Taylor series approximation about $x$ for a function $f : \mathbb{R}^n \rightarrow \mathbb{R}$?

$$f(x + s) =$$

(b) Where does your Taylor approximation achieve its minimum?

(c) Consider $f(x) = 5x^2 + 3x + 1$. How many iterations does Newton’s method (in 1D, as discussed last time) use to converge to the minimum of $f$?

(d) What is the convergence rate for steepest descent in the observed demonstration?

Problem 2: Gauss-Newton

(a) Suppose you want to fit the function $f(t_i, x) = x_0e^{x_1t}$ to some data, say $(t_i, y_i)$ for $i = 1, \ldots, 4$. What function do you want to minimize?

(b) What is the gradient of this function?

(c) What is the difference between a Newton method for this problem and a Gauss-Newton method for this problem?