Numerical Analysis (CS 450)

Worksheet 21

Objectives: **(1)** Use n-dimension Taylor approximation to derive Newton's method **(2)** Understand limitations of steepest-descent methods

Problem 1: Quadratic approximation and Newton

(a) Write down the $O(h^3)$ Taylor series approximation about x for a function $f : \mathbb{R}^n \to \mathbb{R}$?

$f(x + s) =$

(b) Where does your Taylor approximation achieve its minimum?

(c) Consider $f(x) = 5x^2 + 3x + 1$. How many iterations does Newton's method (in 1D, as discussed last time) use to converge to the minimum of f?

(d) What is the convergence rate for steepest descent in the observed demonstration?

Problem 2: Gauss-Newton

(a) Suppose you want to fit the function $f(t_i, \boldsymbol{x}) = x_0 e^{x_1 t}$ to some data, say (t_i, y_i) for $i = 1, \ldots, 4$. What function do you want to minimize?

(b) What is the gradient of this function?

(c) What is the difference between a Newton method for this problem and a Gauss-Newton method for this problem?