

Worksheet 24

Objectives: (1) Be able to build a Lagrange basis (2) Understand properties of different interpolation bases (3) Build intuition on the conditioning of the interpolation problem

Problem 1: Interpolation basics

(a) Which of the following is *not* a good application of interpolation?

- (a) Smoothing the error in noisy data
- (b) Approximating a complicated function by a simple one
- (c) Computing unknown values in between known values on a table
- (d) Replacing a collection of data points by a smooth curve

(b) Suppose you have the function

$$f(x) = 2 - 3x + 4x^2$$

Write this function as a linear combination of three basis functions (you pick). What do your basis functions *span*?

(c) What is the degree of the j -th Lagrange basis function for interpolating n data points?

Problem 2: More Interpolation

(a) Find an expression for the roots (zeros) of the Chebyshev polynomial $T_k(x) = \cos(k \arccos(x))$.

(b) Write down the criteria by which you would choose between sets of interpolation bases and nodes.