Objectives: (1) Be able to build a Lagrange basis (2) Understand properties of different interpolation bases (3) Build intuition on the conditioning of the interpolation problem

Problem 1: Interpolation basics

(a) Which of the following is not a good application of interpolation?
   (a) Smoothing the error in noisy data
   (b) Approximating a complicated function by a simple one
   (c) Computing unknown values in between known values on a table
   (d) Replacing a collection of data points by a smooth curve

(b) Suppose you have the function

\[ f(x) = 2 - 3x + 4x^2 \]

Write this function as a linear combination of three basis functions (you pick). What do your basis functions span?

(c) What is the degree of the \( j \)-th Lagrange basis function for interpolating \( n \) data points?

Problem 2: More Interpolation

(a) Find an expression for the roots (zeros) of the Chebyshev polynomial \( T_k(x) = \cos(k \arccos(x)) \).

(b) Write down the criteria by which you would choose between sets of interpolation bases and nodes.