

Worksheet 25

Objectives: (1) Understand the error result for the interpolation problem (2) Be able to apply piecewise polynomial interpolation (3) Understand conditioning of the quadrature problem

Problem 1: Interpolation

(a) Find an expression for the roots (zeros) of the Chebyshev polynomial $T_k(x) = \cos(k \arccos(x))$.

(b) Legendre and Chebyshev polynomials generate interpolation methods on the interval $[-1, 1]$. Suppose you would like to interpolate on an arbitrary interval $[a, b]$ instead, but you would still like to use Chebyshev interpolation.

Given $f : [a, b] \rightarrow \mathbb{R}$ and the Chebyshev nodes x_i , what would you use as input for Chebyshev interpolation on $[-1, 1]$?

Given the Chebyshev coefficients α_k (i.e. the result of interpolation on $[-1, 1]$), how would you evaluate the interpolant on $[a, b]$?

(c) Write down the criteria by which you would choose between sets of interpolation bases and nodes.