Numerical Analysis (CS 450)
Worksheet 29

Objectives: (1) Know the following terms as applied to ODEs: IVP, order, autonomous, homogeneous (2) Know how to apply stability regions

Problem 1: ODEs

(a) Rewrite the following ODE as an autonomous first-order system:

\[ y''(t) = 5t^2y(t). \]

(b) Which of the following systems of ODEs is stable?

\[
\begin{align*}
y'(t) &= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} y(t) \\
y'(t) &= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} y(t) \\
y'(t) &= \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} y(t) \\
y'(t) &= \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} y(t)
\end{align*}
\]

Problem 2: Euler method

(a) What is the solution to

\[ u' = -u \]

and

\[ u' = -10u \]

and

\[ u' = -100u \]

with \( u(0) = 1 \)? Sketch them.

(b) What size time step does the Euler method need for each of the previous problems?