Numerical Analysis (CS 450)

Worksheet 30

Objectives: (1) Derive Euler's method **(2)** Know how to apply stability regions **(3)** Understand why the 'test ODE' $y' = \lambda y$ works for predicting stability **(4)** Understand the idea behind 'stiffness'

Problem 1: Euler method

(a) What size time step does the (forward) Euler method need for each of these problems?

$$y' = -y, \qquad y' = -10y, \qquad y' = -100y$$

with $y(0) = 1$?

(b) If a time integrator is 'of order 4', what is the order of the first term in its error expansion?

(c) Consider the following two statements:

(1) The ODE is stable.

(2) The numerical method applied to the ODE is stable.

How are (1) and (2) related? Choose the best answer.

(A) (1) $\Rightarrow$ (2) — If (1), then (2).

(B) (1) $\Leftarrow$ (2) — If (2), then (1).

(C) (1) $\Leftrightarrow$ (2) — (1) if and only if (2).

(D) None of these.

(d) Which of the following statements best describes the use of Euler's method for solving a stiff ODE?

(A) Never stable.

(B) Can be stable but is inefficient.

(C) Can be both stable and efficient.

(D) Always stable.