Objectives: (1) Read and write Butcher tableaux (2) Make use of stability region plots (3) Understand terminology, existence, and uniqueness of BVPs.

Problem 1: Butcher tableaux, stability regions

(a) Write a Butcher tableau for forward and backward Euler.

(b) Write a Butcher tableau for the midpoint method:

\[ y_{k+1} = y_k + hf\left(t_k + \frac{h}{2}, y_k + \frac{h}{2}f(t_k, y_k)\right) \]

(c) Write a Butcher tableau for an implicit method using the trapezoidal rule.

(d) Consider a linear, constant-coefficient system of ODEs \( \mathbf{y}' = A\mathbf{y} \) in which \( A \) has an eigenvalue \(-3 + 4i\). Give an approximate step size \( h \) so that forward Euler will be stable for this system (only considering this eigenvalue).