

The Pochoir Stencil Compiler

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Motivation

- Stencil computations are easy to implement using nested loops. But looping implementations suffer from poor cache performance.
- Cache oblivious algorithms are more efficient, but they are difficult to write, especially when parallelism is involved.



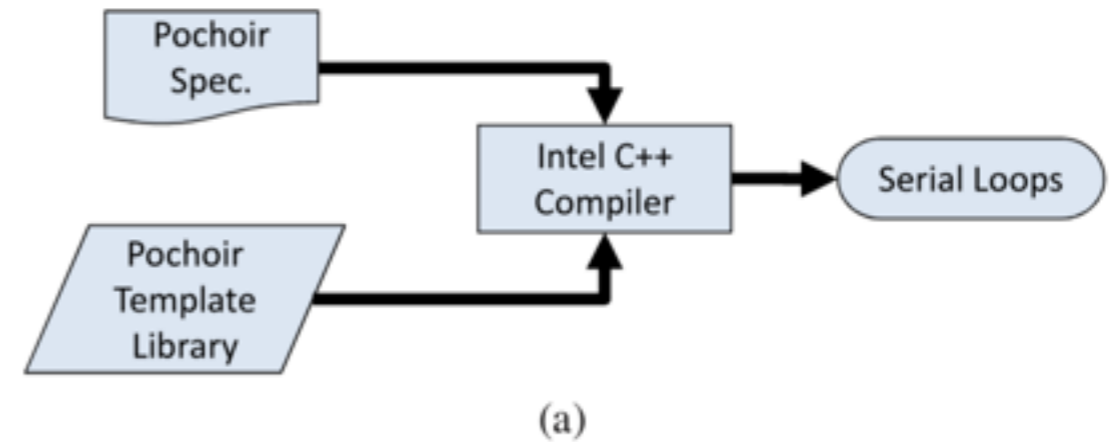
- The Pochoir compiler allows a programmer to write a stencil program in a DSL embedded in C++. The Pochoir compiler then translates it into high-performing **Cilk code** that employs an efficient parallel cache-oblivious algorithm.

What is Pochoir

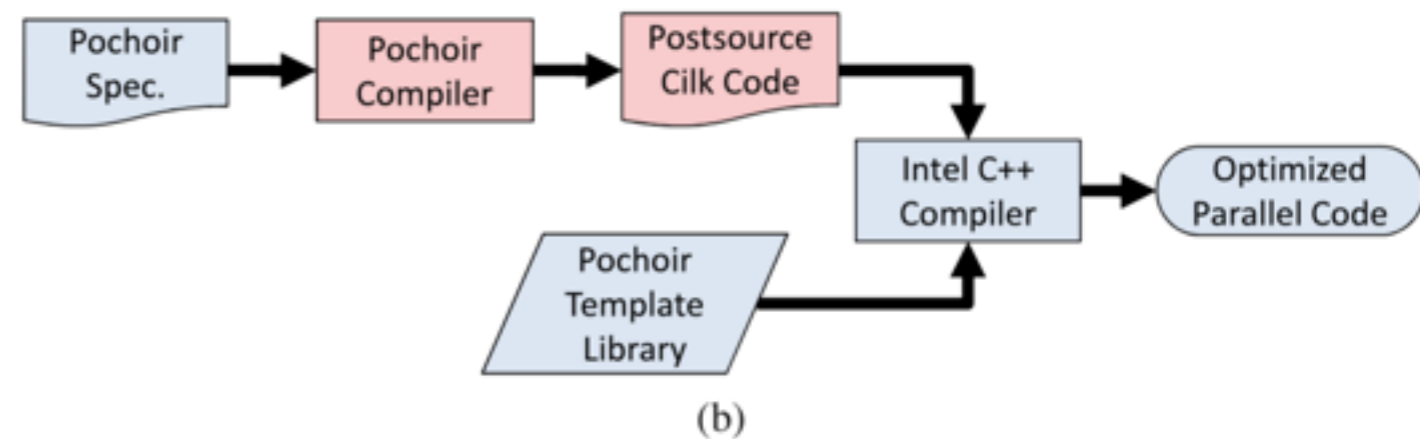
- Pochoir (pronounced as “PO-shwar”; it means “stencil” in French) is a compiler and runtime system for implementing stencil computations on multicore processors.
 1. Pochoir template library
 2. Pochoir compiler

Workflow

- Phase 1:
the programmer uses the normal Intel C++ compiler to compile his or her code with the Pochoir template library. Phase 1 verifies that the programmer's stencil specification is Pochoir compliant.



- Phase 2:
the programmer uses the Pochoir compiler, which acts as a preprocessor to the Intel C++ compiler, to generate optimized multithreaded Cilk code.



Example: periodic 2D heat equation

- 2d heat equation

$$\frac{\partial u_t(x,y)}{\partial t} = \alpha \left(\frac{\partial^2 u_t(x,y)}{\partial x^2} + \frac{\partial^2 u_t(x,y)}{\partial y^2} \right)$$

- Jacobi-style update equation:

$$\begin{aligned} u_{t+1}(x,y) = & u_t(x,y) \\ & + \frac{\alpha \Delta t}{\Delta x^2} (u_t(x-1,y) + u_t(x+1,y) - 2u_t(x,y)) \\ & + \frac{\alpha \Delta t}{\Delta y^2} (u_t(x,y-1) + u_t(x,y+1) - 2u_t(x,y)) . \end{aligned}$$

Example: periodic 2D heat equation

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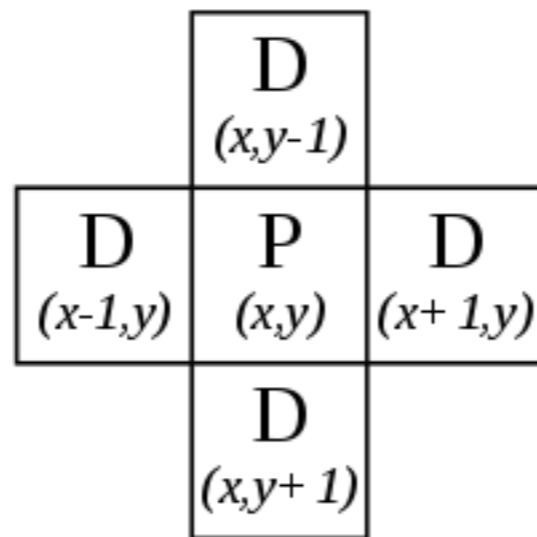
- Simple for loop implementation:

LOOPS(*u*; *ta*, *tb*; *xa*, *xb*; *ya*, *yb*)

```
1  for t = ta to tb - 1
2      parallel for x = xa to xb - 1
3          for y = ya to ya - 1
4              u((t + 1) mod 2, x, y) = u(t mod 2, x, y)
                + CX · (u(t mod 2, (x - 1) mod X, y)
                + u(t mod 2, (x + 1) mod X, y) - 2u(t mod 2, x, y))
                + CY · (u(t mod 2, x, (y - 1) mod Y)
                + u(t mod 2, x, (y + 1) mod Y) - 2u(t mod 2, x, y))
```

Example: periodic 2D heat equation

- **Pochoir_Shape_2D** 2d_five_pt[] = {{0, 0, 0}, {-1, 1, 0}, {-1, 0, 0}, {-1, -1, 0}, {-1, 0, 1}, {-1, 0, -1}} ;



time step
x offset y offset

- **Pochoir_2D** heat(2d_five_pt);

Example: periodic 2D heat equation

function name data array time step spatial coordinates

```
Pochoir_Boundary_2D (heat_bv, array, t, x, y)  
    return array.get(t, mod(x, array.size(1)), mod(y,  
        array.size(0)));  
Pochoir_Boundary_End
```

This construct defines a boundary function called **heat_bv** that will be invoked to supply a value when the stencil computation accesses a point outside the domain of the Pochoir array array.

Example: periodic 2D heat equation

function name data array time step spatial coordinates

Pochoir_Kernel_2D (heat_fn, u, t, x, y)

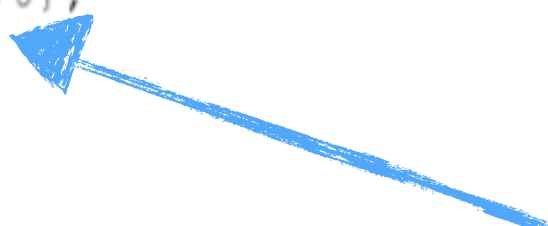
$$u(t+1, x, y) = CX * (u(t, x+1, y) - 2 * u(t, x, y) + u(t, x-1, y)) +$$
$$CY * (u(t, x, y+1) - 2 * u(t, x, y) + u(t, x, y-1)) +$$
$$u(t, x, y);$$

Pochoir_Kernel_End

This construct defines a kernel function named **heat_fn** for updating a stencil on a spatial grid with dim spatial dimensions.

Example: periodic 2D heat equation

```
1  #define mod(r,m) ((r)%(m) + ((r)<0)? (m):0)
2  Pochoir_Boundary_2D(heat_bv, a, t, x, y)
3      return a.get(t,mod(x,a.size(1)),mod(y,a.size(0)));
4  Pochoir_Boundary_End
5
6  int main(void) {
7
8      Pochoir_Shape_2D 2D_five_pt[] = {{1,0,0}, {0,1,0},
9          {0,-1,0}, {0,-1,-1}, {0,0,-1}, {0,0,1}};
10     Pochoir_2D heat(2D_five_pt);
11
12     Pochoir_Array_2D(double) u(X, Y);
13     u.Register_Boundary(heat_bv);
14     heat.Register_Array(u);
15
16     Pochoir_Kernel_2D(heat_fn, t, x, y)
17         u(t+1, x, y) = CX * (u(t, x+1, y) - 2 * u(t, x,
18             y) + u(t, x-1, y)) + CY * (u(t, x, y+1) - 2
19             * u(t, x, y) + u(t, x, y-1)) + u(t, x, y);
20     Pochoir_Kernel_End
21
22     for (int x = 0; x < X; ++x)
23         for (int y = 0; y < Y; ++y)
24             u(0, x, y) = rand();
25
26     heat.Run(T, heat_fn);
27
28     for (int x = 0; x < X; ++x)
29         for (int y = 0; y < Y; ++y)
30             cout << u(T, x, y);
31
32     return 0;
33 }
```



Pochoir_Shape_dimD contains the spatial information. Each of its element has $\text{dim}+1$ integers represent the offset of each memory footprint in the stencil kernel relative to the space-time grid point $\langle t, x, y, \dots \rangle$.

Example: periodic 2D heat equation

```
1  #define mod(r,m) ((r)%(m) + ((r)<0)? (m):0)
2  Pochoir_Boundary_2D(heat_bv, a, t, x, y)
3      return a.get(t,mod(x,a.size(1)),mod(y,a.size(0)));
4  Pochoir_Boundary_End
5
6  int main(void) {
7
8      Pochoir_Shape_2D 2D_five_pt[] = {{1,0,0}, {0,1,0},
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16     Pochoir_Kernel_2D(heat_fn, t, x, y)
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18             y) + u(t, x-1, y)) + CY * (u(t, x, y+1) - 2
19             * u(t, x, y) + u(t, x, y-1)) + u(t, x, y);
20     Pochoir_Kernel_End
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23         for (int y = 0; y < Y; ++y)
24             u(0, x, y) = rand();
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26     heat.Run(T, heat_fn);
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28     for (int x = 0; x < X; ++x)
29         for (int y = 0; y < Y; ++y)
30             cout << u(T, x, y);
31
32     return 0;
33 }
```

The static information about a Pochoir stencil computation, such as the computing kernel, the boundary conditions, and the stencil shape, is stored in a **Pochoir_dimD**

Example: periodic 2D heat equation

```
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2  Pochoir_Boundary_2D(heat_bv, a, t, x, y)
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4  Pochoir_Boundary_End
5
6  int main(void) {
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8      Pochoir_Shape_2D 2D_five_pt[] = {{1,0,0}, {0,1,0},
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10     Pochoir_2D heat(2D_five_pt);
11
12     Pochoir_Array_2D(double) u(X, Y);
13     u.Register_Boundary(heat_bv);
14     heat.Register_Array(u);
15
16     Pochoir_Kernel_2D(heat_fn, t, x, y)
17         u(t+1, x, y) = CX * (u(t, x+1, y) - 2 * u(t, x,
18             y) + u(t, x-1, y)) + CY * (u(t, x, y+1) - 2
19             * u(t, x, y) + u(t, x, y-1)) + u(t, x, y);
20     Pochoir_Kernel_End
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22     for (int x = 0; x < X; ++x)
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24             u(0, x, y) = rand();
25
26     heat.Run(T, heat_fn);
27
28     for (int x = 0; x < X; ++x)
29         for (int y = 0; y < Y; ++y)
30             cout << u(T, x, y);
31
32     return 0;
33 }
```

The boundary function will be invoked to supply a value when the stencil computation accesses a point outside the domain of the Pochoir array array.

Example: periodic 2D heat equation

```
1 #define mod(r,m) ((r)%(m) + ((r)<0)? (m):0)
2 Pochoir_Boundary_2D(heat_bv, a, t, x, y)
3     return a.get(t,mod(x,a.size(1)),mod(y,a.size(0)));
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18            y) + u(t, x-1, y)) + CY * (u(t, x, y+1) - 2
19            * u(t, x, y) + u(t, x, y-1)) + u(t, x, y);
20    Pochoir_Kernel_End
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22    for (int x = 0; x < X; ++x)
23        for (int y = 0; y < Y; ++y)
24            u(0, x, y) = rand();
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33 }
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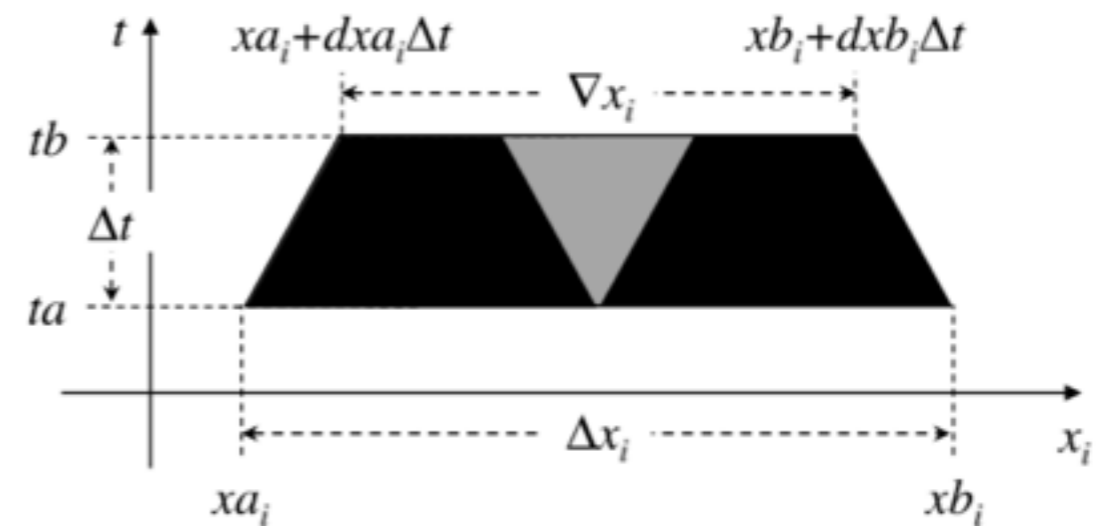
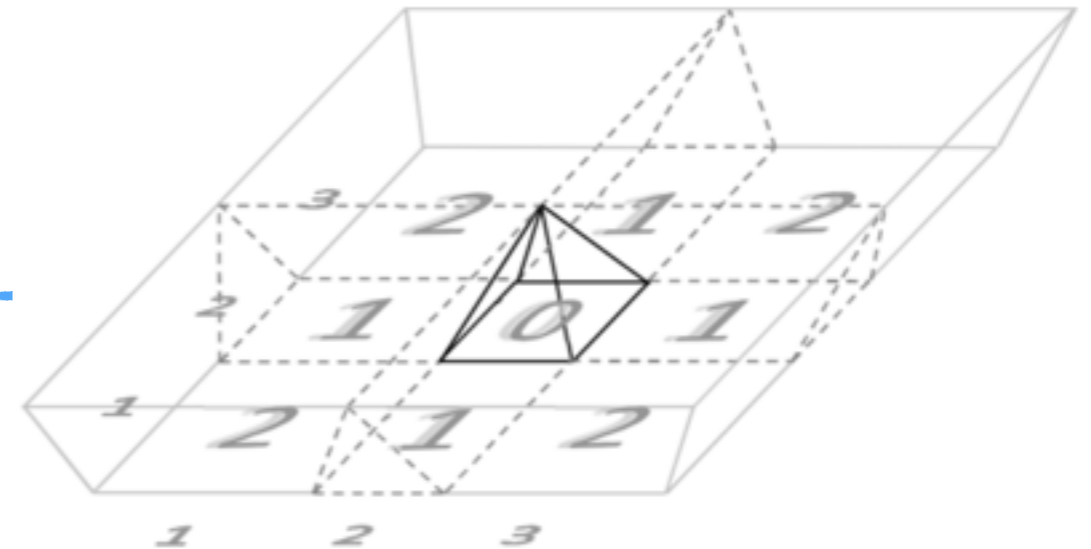
Pochoir_Kernel_dimD arbitrary C++ code for updating a stencil on a spatial grid with dim spatial dimensions.

Trapezoid (zoid) decomposition with hyperspace cut

```

TRAP(u; ta, tb; xa, xb, dxa, dxb; ya, yb, dya, dyb)
1   $\Delta t = tb - ta$ 
2   $\Delta x = \max\{xb - xa, (xb + dxb\Delta t) - (xa + dxa\Delta t)\}$  // Longer x-base
3   $\Delta y = \max\{yb - ya, (yb + dyb\Delta t) - (ya + dya\Delta t)\}$  // Longer y-base
4   $k = 0$  // Try hyperspace cut
5  if  $\Delta x \geq 2\sigma_x\Delta t$ 
6    Trisect the zoid with x-cuts
7     $k += 1$ 
8  if  $\Delta y \geq 2\sigma_y\Delta t$ 
9    Trisect the zoid with y-cuts
10    $k += 1$ 
11  if  $k > 0$ 
12    Assign dependency levels 0, 1, ...,  $k$  to subzoids
13    for  $i = 0$  to  $k$  // for each dependency level  $i$ 
14      parallel for all subzoids
15        ( $ta, tb; xa', xb', dxa', dxb'; ya', yb', dya', dyb'$ )
16        with dependency level  $i$ 
17        TRAP( $ta, tb; xa', xb', dxa', dxb'; ya', yb', dya', dyb'$ )
18  elseif  $\Delta t > 1$  // time cut
19    // Recursively walk the lower zoid and then the upper
20    TRAP( $ta, ta + \Delta t/2; xa, xb, dxa, dxb; ya, yb, dya, dyb$ )
21    TRAP( $ta + \Delta t/2, tb; xa + dxa\Delta t/2, xb + dxb\Delta t/2, dxa, dxb;$ 
22     $ya + dya\Delta t/2, yb + dyb\Delta t/2, dya, dyb$ )
23  else // base case
24    for  $t = ta$  to  $tb - 1$ 
25      for  $x = xa$  to  $xb - 1$ 
26        for  $y = ya$  to  $yb - 1$ 
27           $u((t+1) \bmod 2, x, y) = u(t \bmod 2, x, y)$ 
28             $+ CX \cdot (u(t \bmod 2, (x-1) \bmod X, y)$ 
29               $+ u(t \bmod 2, (x+1) \bmod X, y) - 2u(t \bmod 2, x, y))$ 
30             $+ CY \cdot (u(t \bmod 2, x, (y-1) \bmod Y)$ 
31               $+ u(t \bmod 2, x, (y+1) \bmod Y) - 2u(t \bmod 2, x, y))$ 
32     $xa += dxa$ 
33     $xb += dxb$ 
34     $ya += dya$ 
35     $yb += dyb$ 

```

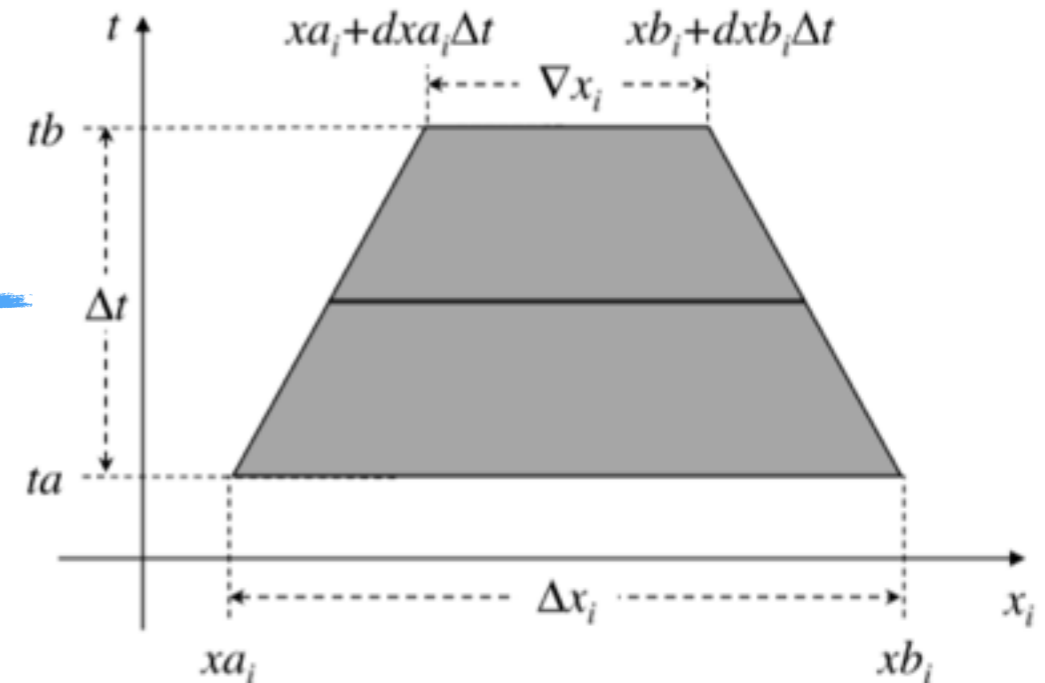


Trapezoid (zoid) decomposition with hyperspace cut

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4   $k = 0$  // Try hyperspace cut
5  if  $\Delta x \geq 2\sigma_x\Delta t$ 
6    Trisect the zoid with x-cuts
7     $k += 1$ 
8  if  $\Delta y \geq 2\sigma_y\Delta t$ 
9    Trisect the zoid with y-cuts
10    $k += 1$ 
11  if  $k > 0$ 
12    Assign dependency levels  $0, 1, \dots, k$  to subzoids
13    for  $i = 0$  to  $k$  // for each dependency level  $i$ 
14      parallel for all subzoids
15        ( $ta, tb; xa', xb', dxa', dxb'; ya', yb', dya', dyb'$ )
16        with dependency level  $i$ 
17        TRAP( $ta, tb; xa', xb', dxa', dxb'; ya', yb', dya', dyb'$ )
18  elseif  $\Delta t > 1$  // time cut
19    // Recursively walk the lower zoid and then the upper
20    TRAP( $ta, ta + \Delta t/2; xa, xb, dxa, dxb; ya, yb, dya, dyb$ )
21    TRAP( $ta + \Delta t/2, tb; xa + dxa\Delta t/2, xb + dxb\Delta t/2, dxa, dxb;$ 
22     $ya + dya\Delta t/2, yb + dyb\Delta t/2, dya, dyb$ )
23  else // base case
24    for  $t = ta$  to  $tb - 1$ 
25      for  $x = xa$  to  $xb - 1$ 
26        for  $y = ya$  to  $yb - 1$ 
27           $u((t+1) \bmod 2, x, y) = u(t \bmod 2, x, y)$ 
28             $+ CX \cdot (u(t \bmod 2, (x-1) \bmod X, y))$ 
29               $+ u(t \bmod 2, (x+1) \bmod X, y) - 2u(t \bmod 2, x, y))$ 
30               $+ CY \cdot (u(t \bmod 2, x, (y-1) \bmod Y)$ 
31                 $+ u(t \bmod 2, x, (y+1) \bmod Y) - 2u(t \bmod 2, x, y))$ 
32     $xa += dxa$ 
33     $xb += dxb$ 
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13    for  $i = 0$  to  $k$  // for each dependency level  $i$ 
14      parallel for all subzoids
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16        with dependency level  $i$ 
17        TRAP(ta, tb; xa', xb', dxa', dxb'; ya', yb', dya', dyb')
18  elseif  $\Delta t > 1$  // time cut
19    // Recursively walk the lower zoid and then the upper
20    TRAP(ta, ta +  $\Delta t/2$ ; xa, xb, dxa, dxb; ya, yb, dya, dyb)
21    TRAP(ta +  $\Delta t/2$ , tb; xa +  $dxa\Delta t/2$ , xb +  $dxb\Delta t/2$ , dxa, dxb;
22    ya +  $dya\Delta t/2$ , yb +  $dyb\Delta t/2$ , dya, dyb)
23  else // base case
24    for  $t = ta$  to  $tb - 1$ 
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30         $+ CY \cdot (u(t \bmod 2, x, (y - 1) \bmod Y)$ 
31         $+ u(t \bmod 2, x, (y + 1) \bmod Y) - 2u(t \bmod 2, x, y))$ 
32     $xa += dxa$ 
33     $xb += dxb$ 
34     $ya += dya$ 
35     $yb += dyb$ 
```

Base case ($\Delta t = 1$)

Coarsening of Base Cases

- Although trapezoidal decomposition reduces cache-miss rates, overall performance can suffer from function-call overhead unless the base case of the recursion is coarsened.
- Solution: reduce the overhead of function(kernel) calls by **coarsening of base cases**.
 - For 2D problems, Pochoir stops the recursion at 100 × 100 space chunks with 5 time steps.
 - For 3D problems, the recursion stops at 1000 × 3 × 3 with 3 time steps.
 - Higher dimensions?

Trapezoid (zoid) decomposition with hyperspace cut

```
TRAP(u; ta, tb; xa, xb, dxa, dxb; ya, yb, dya, dyb)
1   $\Delta t = tb - ta$ 
2   $\Delta x = \max \{xb - xa, (xb + dxb\Delta t) - (xa + dxa\Delta t)\}$  // Longer x-base
3   $\Delta y = \max \{yb - ya, (yb + dyb\Delta t) - (ya + dya\Delta t)\}$  // Longer y-base
4   $k = 0$  // Try hyperspace cut
5  if  $\Delta x \geq 2\sigma_x\Delta t$ 
6    Trisect the zoid with x-cuts
7     $k += 1$ 
8  if  $\Delta y \geq 2\sigma_y\Delta t$ 
9    Trisect the zoid with y-cuts
10    $k += 1$ 
11  if  $k > 0$ 
12    Assign dependency levels  $0, 1, \dots, k$  to subzoids
13    for  $i = 0$  to  $k$  // for each dependency level  $i$ 
14      parallel for all subzoids
15        ( $ta, tb; xa', xb', dxa', dxb'; ya', yb', dya', dyb'$ )
16        with dependency level  $i$ 
17        TRAP( $ta, tb; xa', xb', dxa', dxb'; ya', yb', dya', dyb'$ )
18  elseif  $\Delta t > 1$  // time cut
19    // Recursively walk the lower zoid and then the upper
20    TRAP( $ta, ta + \Delta t/2; xa, xb, dxa, dxb; ya, yb, dya, dyb$ )
21    TRAP( $ta + \Delta t/2, tb; xa + dxa\Delta t/2, xb + dxb\Delta t/2, dxa, dxb;$ 
22     $ya + dya\Delta t/2, yb + dyb\Delta t/2, dya, dyb$ )
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32     $xa += dxa$ 
33     $xb += dxb$ 
34     $ya += dya$ 
35     $yb += dyb$ 
```

- For 2D problems, Pochoir stops the recursion at 100×100 space chunks with 5 time steps.
- For 3D problems, the recursion stops at $1000 \times 3 \times 3$ with 3 time steps.

Handling boundary conditions with code cloning

- Pochoir compiler generates two code clones of the kernel function:
 1. a slower **boundary clone**: the boundary clone is used for boundary zoids: those that contain at least one point whose computation requires an off-grid access.
 2. a faster **interior clone**: the interior clone is used for interior zoids: those all of whose points can be updated without indexing off the edge of the grid.

Loop Indexing

Two ways to generate the interior clone of the kernel function.
-split-pointer by default. User can decide by command-line option.

- -split-macro-shadow

```
1 Pochoir_Kernel_1D(heat_1D_fn, t, i)
2   a(t+1, i) = 0.125 * (a(t, i-1) + 2 * a(t, i) +
3     a(t, i+1));
4 Pochoir_Kernel_End
```

(a)

```
1 /* a.interior() is a function to dereference the
2    value without checking boundary conditions */
3 #define a(t, i) a.interior(t, i)
4 Pochoir_Kernel_1D(heat_1D_fn, t, i)
5   a(t + 1, i) = 0.125 * (a(t, i - 1) + 2 * a(t, i
6     ) + a(t, i + 1));
7 Pochoir_Kernel_End
8 #undef a(t, i)
```

(b)

- -split-pointer

```
1 Pochoir_Kernel_1D(heat_1D_fn, t, i)
2 /* The base address of the Pochoir array 'a' */
3 double *a_base = a.data();
4 /* Pointers to be used in the innermost loop */
5 double *iter0, *iter1, *iter2, *iter3;
6 /* Total size of the Pochoir array 'a' */
7 const int l_a_total_size = a.total_size();
8 int gap_a_0;
9 const int l_stride_a_0 = a.stride(0);
10 for (int t = ta; t < tb; ++t) {
11   double * baseIter_1;
12   double * baseIter_0;
13   baseIter_0 = a_base + ((t + 1) & 0xb) *
14     l_a_total_size + (l_grid.xa[0]) *
15     l_stride_a_0;
16   baseIter_1 = a_base + ((t) & 0xb) *
17     l_a_total_size + (l_grid.xa[0]) *
18     l_stride_a_0;
19   iter0 = baseIter_0 + (0) * l_stride_a_0;
20   iter1 = baseIter_1 + (-1) * l_stride_a_0;
21   iter2 = baseIter_1 + (0) * l_stride_a_0;
22   iter3 = baseIter_1 + (1) * l_stride_a_0;
23   for (int i = l_grid.xa[0]; i < l_grid.xb[0];
24     ++i, ++iter0, ++iter1, ++iter2, ++iter3) {
25     (*iter0) = 0.125 * ((*iter1) + 2 * (*iter2) +
26       (*iter3));
27   }
28 }
29 Pochoir_Kernel_End
```

(c)

Loop Indexing

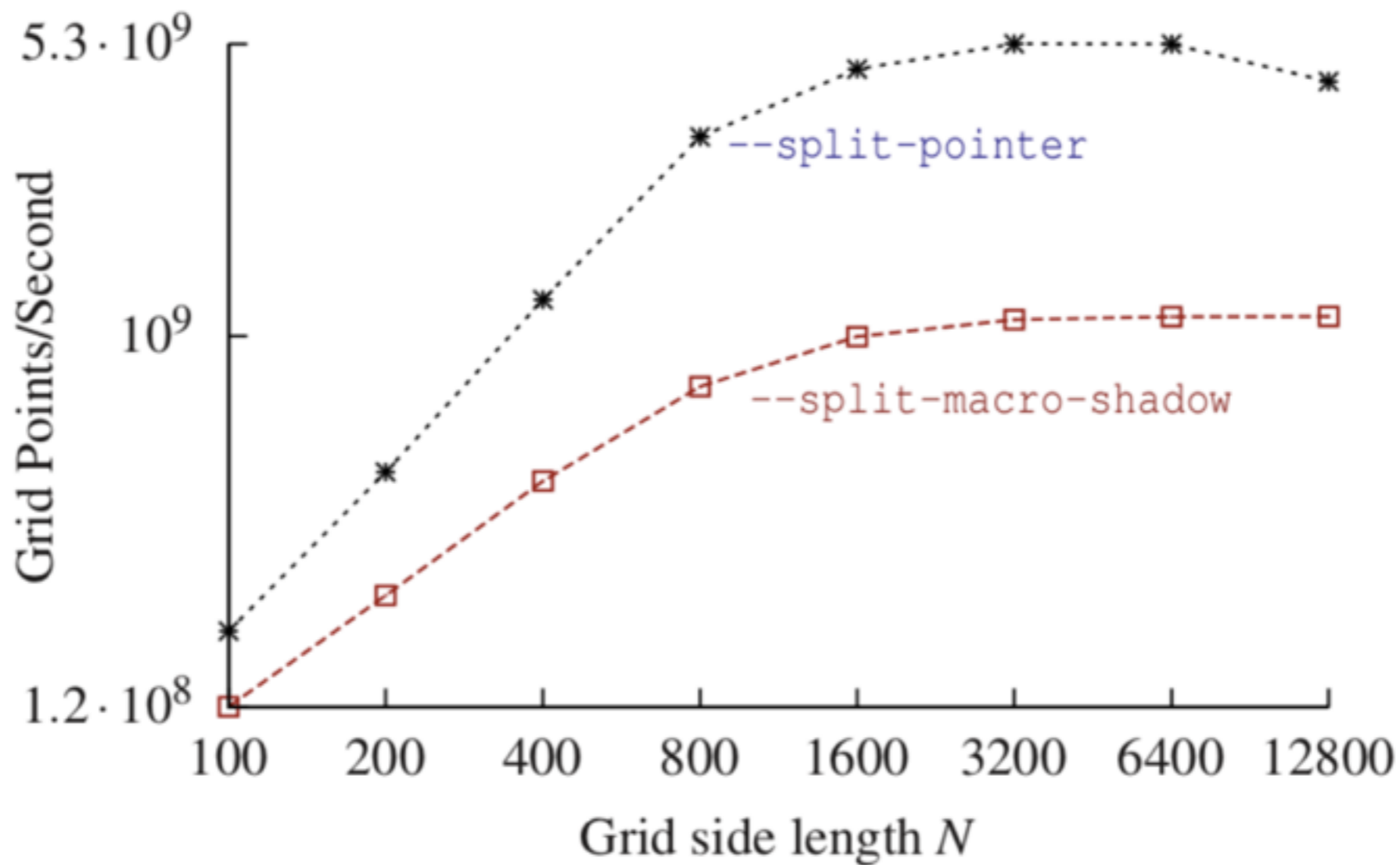
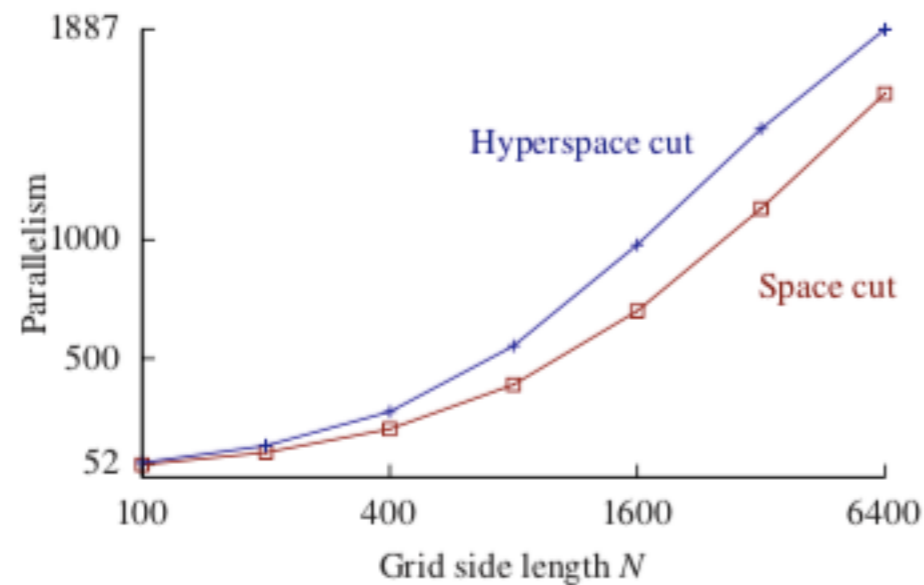
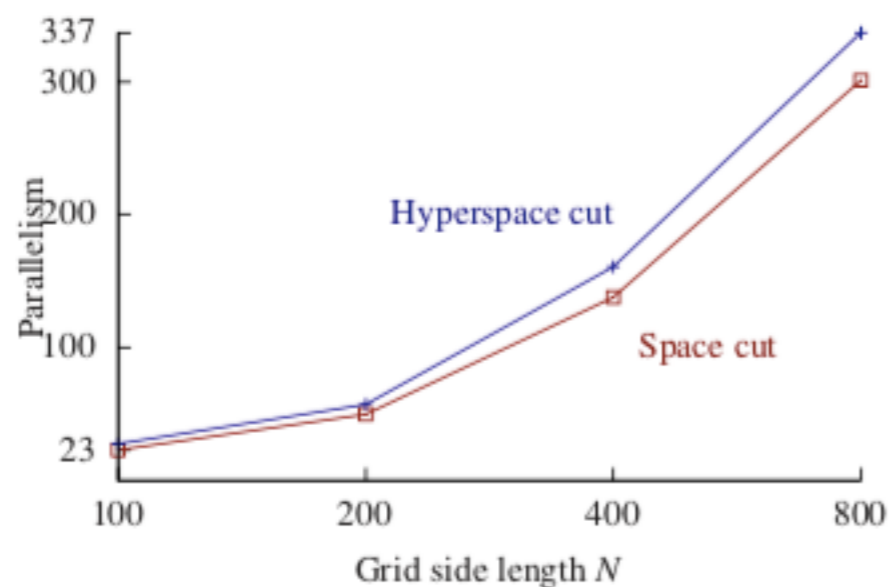


Figure 13: The performance of different loop-index optimizations on a 2D heat equation on torus. The grid is N^2 with 1000 time steps.

Parallelism and cache miss ratio

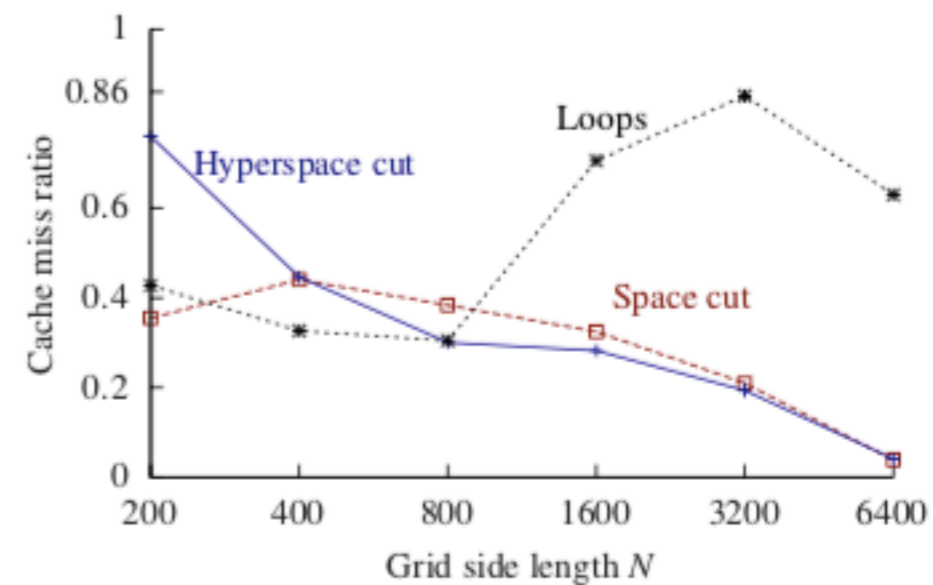


(a)

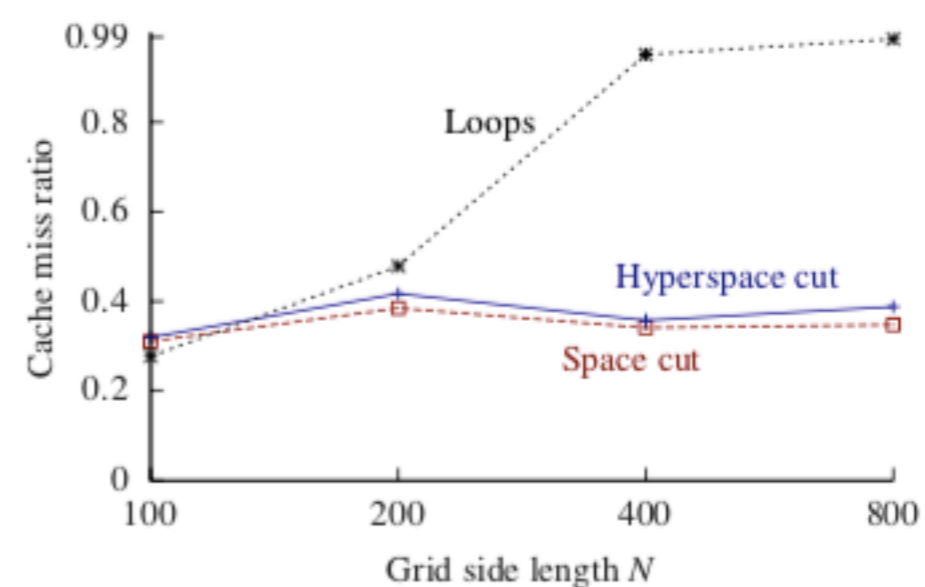


(b)

Figure 9: Parallelism comparison on two benchmarks between TRAP, which employs hyperspace cuts, and STRAP, which uses serial space cuts. Measurements are of code without base-case coarsening. (a) 2D nonperiodic heat equation. Space-time size is $1000N^2$. (b) 3D nonperiodic wave equation. Space-time size is $1000N^3$.



(a)



(b)

Figure 10: Cache-miss ratios for two benchmarks using TRAP, STRAP, and a parallel-loop algorithm. The cache-miss ratio is the ratio of the cache misses to the number of memory references. Measurements are of code without base-case coarsening. (a) 2D nonperiodic heat equation. Space-time is $1000N^2$. (b) 3D nonperiodic wave equation. Space-time is $1000N^3$.

Benchmark

- Heat: heat equation on a 2D grid, a 2D torus, and a 4D grid;
- Life: Conway's game of Life (Life)
- Wave: 3D finite-difference wave equation
- LBM: lattice Boltzmann method (LBM)
- RNA: RNA secondary structure prediction
- PSA: pairwise sequence alignment
- LCS: longest common subsequence
- APOP: American put stock option pricing (APOP)

Benchmark

Pochoir performance on an Intel Core i7 (Nehalem) machine

<i>Benchmark</i>	<i>Dims</i>	<i>Grid size</i>	<i>Time steps</i>	<i>Pochoir</i>			<i>Serial loops</i>		<i>12-core loops</i>	
				<i>1 core</i>	<i>12 cores</i>	<i>speedup</i>	<i>time</i>	<i>ratio</i>	<i>time</i>	<i>ratio</i>
Heat	2	16,000 ²	500	277s	24s	11.5	612s	25.5	149s	6.2
Heat	2p	16,000 ²	500	281s	24s	11.7	1,647s	68.6	248s	10.3
Heat	4	150 ⁴	100	154s	54s	2.9	433s	8.0	104s	1.9
Life	2p	16,000 ²	500	345s	28s	12.3	2,419s	86.4	332s	11.9
Wave	3	1,000 ³	500	3,082s	447s	6.9	3,170s	7.1	1,071s	2.4
LBM	3	100 ² × 130	3,000	345s	68s	5.1	304s	4.5	220s	3.2
RNA	2	300 ²	900	90s	20s	4.5	121s	6.1	26s	1.3
PSA	1	100,000	200,000	105s	18s	5.8	432s	24.0	77s	4.3
LCS	1	100,000	200,000	57s	9s	6.3	105s	11.7	27s	3.0
APOP	1	2,000,000	10,000	43s	4s	10.7	515s	128.8	48s	12.0

- **serial loops:** a serial for loop implementation running on one core
- **12-core loops:** a parallel cilk_for loop implementation running on 12 cores.
- **ratio:** indicates how much slower the looping implementation is than the 12-core Pochoir implementation
- **p** in dims means periodic

Comparison

- The Berkeley **autotuner** focuses on optimizing the performance of stencil kernels by automatically selecting tuning parameters. Their work serves as a good benchmark for the maximum possible speedup one can get on a stencil.
- 7-point stencil and a 27-point stencil on a 2583 grid with “ghost cells”
- “Unfortunately, we were unable to reproduce the reported results from — presumably because there were too many differences in hardware, compilers, and operating system ”

Comparison

	<i>Berkeley</i>	<i>Pochoir</i>
CPU	Xeon X5550	Xeon X5650
Clock	2.66GHz	2.66 GHz
cores/socket	4	6
Total # cores	8	12
Hyperthreading	Enabled	Disabled
L1 data cache/core	32KB	32KB
L2 cache/core	256KB	256KB
L3 cache/socket	8MB	12 MB
Peak computation	85 GFLOPS	120 GFLOPS
Compiler	icc 10.0.0	icc 12.0.0
Linux kernel		2.6.32
Threading model	Pthreads	Cilk Plus
3D 7-point 8 cores	2.0 GStencil/s 15.8 GFLOPS	2.49 GStencil/s 19.92 GFLOPS
3D 27-point 8 cores	0.95 GStencil/s 28.5 GFLOPS	0.88 GStencil/s 26.4 GFLOPS

Conclusion

- Easier to write parallel cache efficient stencil program.
- Two phases methodology
- Trapezoid decomposition with hyperspace cut

Some questions

- Compare with hand-written parallel cache efficient algorithms?
- Doesn't support irregularly shaped domains.
- Performance decomposition?
- Performance of dimension > 3 ?
- Scalability